CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-10

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. A rational number equivalent to a rational number $\frac{7}{19}$ is [1]

(a) $\frac{17}{119}$ (b) $\frac{14}{57}$

(c) $\frac{21}{38}$ (d) $\frac{21}{57}$

Ans : (d) $\frac{21}{57}$

$$\text{Simplest form of } \frac{17}{119} = \frac{17}{119}$$

$$\text{Simplest form of } \frac{14}{57} = \frac{14}{57}$$

$$\text{Simplest form of } \frac{21}{38} = \frac{21}{38}$$

$$\text{Simplest form of } \frac{21}{57} = \frac{7}{19}$$

2. If $x = -2$ and $x^2 + y^2 + 3xy = -5$, then find [1]

(a) -2 (b) 3

(c) -4 (d) 9

Ans : (b) 3

Value of

$$\begin{aligned} x^2 + y^2 + 3xy &= -5 \text{ and } x = -2 \\ (-2)^2 + y^2 - 6y &= -5 \\ y^2 - 6y + 9 &= 0 \\ (y-3)^2 &= 0 \\ y - 3 &= 0 \\ y &= 3 \end{aligned}$$

3. A point whose abscissa is -3 and ordinate is 2 lies in[1]

(a) first quadrant (b) second quadrant

(c) third quadrant (d) fourth quadrant

Ans : (b) second quadrant

Since the abscissa is negative and the ordinate is positive. So, the point lies in second quadrant.

4. If $(3, -2)$ is a solution of the equation $3x - py - 7 = 0$, then the value of p is [1]

(a) -1 (b) 1

(c) $-\frac{13}{3}$ (d) 2

Ans : (a) -1

$$\begin{aligned} 3(3) - p(-2) - 7 &= 0 \\ 2 + 2p &= 0 \\ p &= -1 \end{aligned}$$

5. Two distinct lines [1]

(a) always intersect.

(b) always either intersect or are parallel.

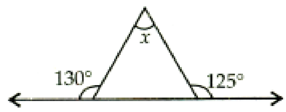
(c) always have two common points.

(d) are always parallel.

Ans : (b) always either intersect or are parallel.

Two distinct lines can either intersect or parallel. It may be possible that point of intersection is either 1 or infinite which means they coincide.

6. Find the value of x. [1]



(a) 70° (b) 75°

(c) 60° (d) 65°

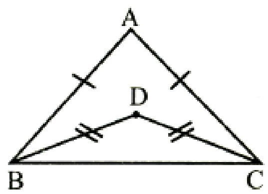
Ans : (b) 75°

$$x + (180° - 130°) + (180° - 125°) = 180°$$

[Angle sum property of a triangle]

$$\begin{aligned} x + 50° + 55° &= 180° \\ x &= 75° \end{aligned}$$

7. In the given figure, the ratio $\angle ABD : \angle ACD$ is [1]



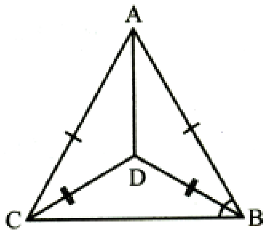
(a) 1 : 1 (b) 2 : 1

(c) 1 : 2 (d) 2 : 3

Ans : (a) 1 : 1

Join A to D,

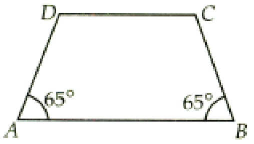
then, $\Delta ABD \cong \Delta ACD$ [SSS Criterion]



$\angle ABD = \angle ACD$ [CPCT]

Hence, $\angle ABD : \angle ACD = 1:1$

8. In the given figure $AB \parallel CD$, then measure of $\angle C$ is [1]



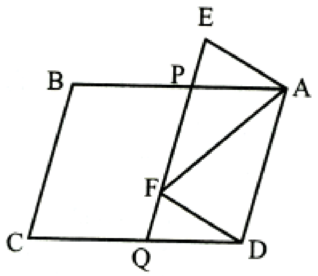
(a) 65° (b) 115°
(c) 135° (d) 125°

Ans : (b) 115°

$AB \parallel DC$ and $\angle B + \angle C = 180°$

$\angle C = 180° - 65° = 115°$

9. In figure $ABCD$ and $AEFD$ are two parallelograms, then the ratio of ar(ΔPEA) to the ar(ΔQFD). [1]



(a) 1 : 4 (b) 1 : 3
(c) 1 : 2 (d) 1 : 1

Ans : (d) 1 : 1

In triangles PEA and QFD, we have

$\angle APE = \angle DQF$ (Corresponding angles)

$AE = DF$ (Opposite sides of $\parallel$ gm $AEFD$)

$\angle AEP = \angle DEQ$ (Corresponding angles)

Hence, $\Delta PEA \cong \Delta QFD$

(ASA congruence criterion)

As congruent triangles have equal area,

Hence, area(ΔPEA) = area(ΔQFD)

Hence, $\frac{\text{area}(\Delta PEA)}{\text{area}(\Delta QFD)} = \frac{1}{1} = 1:1$

10. The line joining the centre of a circle to the midpoint of a chord is always [1]
(a) parallel to the chord
(b) perpendicular to the chord
(c) equal to the chord
(d) tangent to the chord

Ans : (b) perpendicular to the chord

(Q.11-Q.15) Fill in the blanks :

11. The construction of a triangle ABC, given that $BC = 3\,\text{cm}$, $\angle C = 60°$ is possible when difference of AB and AC is equal to .... cm. [1]

Ans : 2.8 cm

A triangle can be constructed when difference of two of its sides is less than the third side.

12. The value of semi-perimeter of an equilateral triangle having area $4\sqrt{3}$ cm^2 is .... cm. [1]

Ans : 6 cm

Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$$4\sqrt{3} = \frac{\sqrt{3}}{4}a^2$$

$$a^2 = 16$$

$$a = 4\,\text{cm}$$

$$\text{Semi-perimeter} = \frac{4+4+4}{2} = \frac{12}{2} = 6\,\text{cm}$$

or

Area of a triangle with perimeter 42 cm and length of two sides 18 cm and 10 cm is given by ....

Ans : $21\sqrt{11}$ cm^2

13. The volume of a sphere is .... to two-thirds the volume of a cylinder of the same height and diameter. [1]

Ans : equal

14. The range of the data 15, 20, 6, 5, 30, 35, 93, 34, 91, 17, 83, is .... [1]

Ans : $93 - 5 = 88$

15. An experiment is called a .... experiment if all the possible outcomes are pre-decided. [1]

Ans : Random

(Q.16-Q.20) Answer the following :

16. Find the coefficient of x^2 in $(3x + x^3)\left(x\frac{1}{x}\right)$ [1]

SOLUTION :

Let $I = (3x + x^3)\left(x + \frac{1}{x}\right)$

$$= 3x \times x \times 3x \times \frac{1}{x} + x^3 \times x \times x^3 \times \frac{1}{x}$$

$$= 3x^2 + 3 + 4x^4 + x^2 = x^4 + 4x^2 + 3$$

So, the coefficient of x^2 is 4

17. In which quadrant $(6, -4)$ will lie? [1]

SOLUTION :

In a point $(6, -4)$, x coordinate is positive and y-coordinate is negative, so it lies in IV quadrant.

18. Euclid divided the 'elements' into how many books?[1]

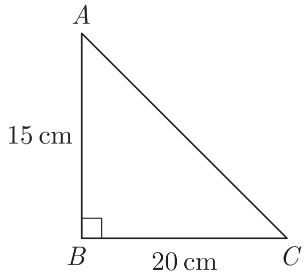
SOLUTION :

Euclid divides his famous treatise 'elements' into 13 chapters, each called a book.

19. Find the area of right triangle in which the sides containing the right angle measure 20 cm and 15 cm [1]

SOLUTION :

$$\text{Area of the right triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$= \frac{1}{2} \times 20 \times 15 \text{ cm}^2$$
$$= 150 \text{ cm}^2$$



20. What is the mean of prime numbers between 20 and 30. [1]

SOLUTION :

Prime numbers between 20 and 30 are 23 and 29.

$$\text{Mean of 23 and 29} = \frac{23+29}{2} = \frac{52}{2} = 26$$

Section B

21. Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, if $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$. [2]

SOLUTION :

We have :

Let $I \; \frac{6}{\sqrt{5}-\sqrt{3}}$

$$= \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$
$$= \frac{6(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= \frac{6(\sqrt{5}+\sqrt{3})}{5-3}$$
$$= \frac{6(\sqrt{5}+\sqrt{3})}{2}$$
$$= 3(\sqrt{5}+\sqrt{3})$$
$$= 3(2.236+1.732)$$
$$= 3(3.968) = 11.904$$

or

Simplify : $\frac{8(5\sqrt{2}+1)}{(\sqrt{2}+1)^2-(\sqrt{2}-1)^2}$

and express as rational denominator.

SOLUTION :

We have $\frac{8(5\sqrt{2}+1)}{(\sqrt{2}+1)^2-(\sqrt{2}-1)^2}$

$$= \frac{8(5\sqrt{2}+1)}{2+1+2\sqrt{2}-2-1+2\sqrt{2}}$$
$$= \frac{8(5\sqrt{2}+1)}{4\sqrt{2}}$$
$$= \frac{2(5\sqrt{2}+1)}{\sqrt{2}}$$
$$= \frac{2\sqrt{2}(5\sqrt{2}+1)}{\sqrt{2}\times\sqrt{2}}$$
$$= \sqrt{2}(5\sqrt{2}+1)$$
$$= 10+\sqrt{2}$$

22. Find the value of k for the given below equation if $x = 1$ and $y = 1$ is its solution. $9\,kx + 12\,ky = 63$ [2]

SOLUTION :

Given equation is $9k\,x + 12k\,y = 63$

On putting $x = 1$ and $y = 1$ in this equation, we get

$$9\,k(1) + 12\,k(1) = 63$$
$$\Rightarrow \quad 9k + 12k = 63$$
$$21k = 63$$
$$k = \frac{63}{21} = 3$$

Hence, the required value of k is 3.

or

Show that $x+3$ is a factor of $69 + 11x - x^2 + x^3$.

SOLUTION :

Let $p(x) = 69 + 11x - x^2 + x^3$

Now, $p(-3) = 69 + 11(-3) - (-3)^2 + (-3)^3$

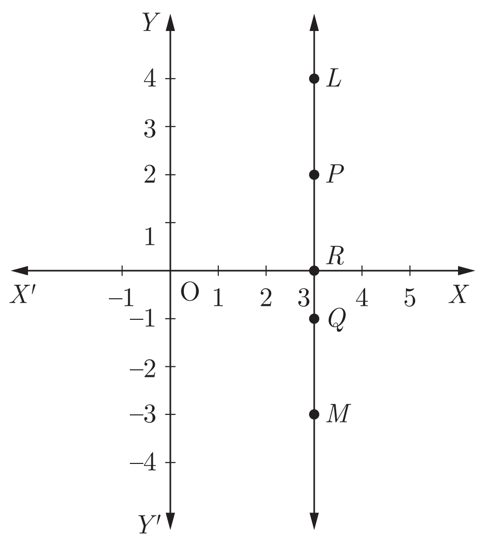
$$= 69 - 33 - 9 - 27$$
$$= 36 - 36 = 0$$

Since, $p(-3) = 0$, therefore, by factor theorem $(x+3)$ is a factor of $p(x)$.

23. In the figure, LM is a line parallel to the y-axis at a distance of 3 units.

(i) What are the coordinates of P, R and Q ?

(ii) What is the difference between the abscissa of the points L and M ? [2]



SOLUTION :

(i) Coordinates of P, R and Q respectively are (3, 2), (3, 0) and (3, –1).

(ii) We have, coordinates of L are (3, 4) coordinates of M are (3, –3).

So, difference between the abscissa of the points L and $M = 3 - 3 = 0$.

24. Find the supplement of $\frac{3}{5}$ of a right angle. [2]

SOLUTION :

Given, angle $= \frac{3}{5}$ of a right angle

$$= \frac{3}{5} \times 90^\circ$$

$$= 3 \times 18^\circ = 54^\circ$$

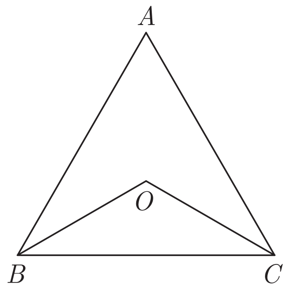
Supplement of 54° = An angle of measure $(180^\circ - 54^\circ)$

= An angle of measure 126°

Hence, the required angle is 126°.

or

In the given figure, ABC is an equilateral triangle. The bisectors of $\angle ABC$ and $\angle ACB$ meet at O. Find the measure of $\angle BOC$.



SOLUTION :

$$\angle BOC + \angle OBC + \angle OCB = 180^\circ$$

[Angles of a triangle]

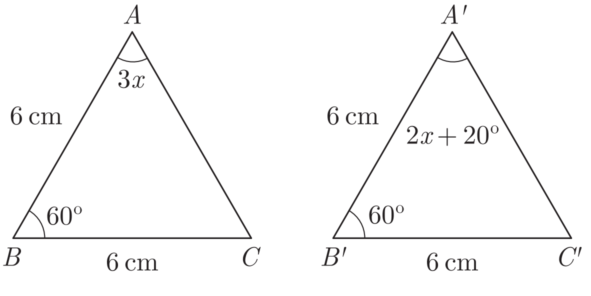
$$\Rightarrow \quad \angle BOC + \frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB = 180^\circ$$

$$\angle BOC + 30^\circ + 30^\circ = 180^\circ$$

[$\because$ ΔABC is equilateral]

$$\angle BOC = 180^\circ - 60^\circ = 120^\circ$$

25. In the given figure, find the measure of $\angle B'A'C'$. [2]



SOLUTION :

In ΔABC and $\Delta A'B'C'$,

$AB = A'B' = 6$ cm [Given]

$\angle ABC = \angle A'B'C' = 60^\circ$ [Given]

and $BC = B'C' = 6$ cm [Given]

$\therefore$ $\Delta ABC \cong \Delta A'B'C'$

[By SAS congruence rules]

Then, $\angle BAC = \angle B'A'C'$ [By CPCT]

$\Rightarrow$ $3x = 2x + 20^\circ$

$3x - 2x = 20^\circ$

$x = 20^\circ$

$\therefore$ $\angle B'A'C' = 2x + 20^\circ$

$= 2 \times 20^\circ + 20^\circ = 60^\circ$

26. The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4, then find the ratio of their curved surface areas. [2]

SOLUTION :

Given, diameters of two cones are equal. So, radii of these cones are also equal. Let r be the radius of each cone and slant height be $5x$ and $4x$.

We know that,

curved surface area of a cone $= \pi rl$

Then curved surface area of first cone $= \pi r \times 5x$

and curved surface area of second cone $= \pi r \times 4x$

$$\therefore \quad \text{Required Ratio} = \frac{\pi r \times 5x}{\pi r \times 4x} = 5:4$$

Section C

27. The linear equation that converts Fahrenheit (F) to Celsius (C), is given by the relation $C = \frac{5F - 160}{9}$. [3]

(i) If the temperature is 86°F, then what is the temperature in Celsius ?

(ii) If the temperature is 35°C, then what is the temperature in Fahrenheit ?

(iii) If the temperature is 0°C, then what is the temperature in Fahrenheit and if temperature is 0°F, then what is the temperature in Celsius ?

SOLUTION :

(i) Here, $F = 86^\circ$

Then, $$C = \frac{5 \times 86 - 160}{9} = \frac{430 - 160}{9}$$

$$= \frac{270}{9} = 30^\circ$$

(ii) Here, $C = 35^\circ$

Then, $$35 = \frac{5F - 160}{9}$$

$$35 \times 9 = 5F - 160$$

$$315 + 160 = 5F$$

$$5F = 475$$

$$F = \frac{475}{5} = 95^\circ$$

(iii) When temperature is 0°C celsius

Then, $$0 = \frac{5F - 160}{9}$$

$$5F = 160$$

$$F = \frac{160}{5} = 32^\circ$$

When temperature is 0° Fahrenheit

Then, $$C = \frac{5 \times 0 - 160}{9}$$

$$= \left(\frac{-160}{9}\right)^{\circ}$$

or

Solve : $4x - 18 = 3y$, $6x + 7y - 4 = 0$.

SOLUTION :

The given equations are :

$$4x - 3y = 18 \quad ...(1)$$

$$6x + 7y = 4 \quad ...(2)$$

Multiplying (1) by 3 and (2) by 2, we get

$$12x - 9y = 54 \quad ...(3)$$

$$12x + 14y = 8 \quad ...(4)$$

Subtracting (4) from (3), we get

$$-23y = 46$$

$$y = -2$$

Substituting $y = -2$ in (1), we get

$$4x - 3 \times (-2) = 18$$

$$\Rightarrow \quad 4x + 6 = 18$$

$$4x = 12$$

$$x = 3$$

Hence, $x = 3$ and $y = -2$ is the solution of the given equations.

28. Prove that, if a transversal intersects two lines, such that pair of alternate interior angles is equal, then the two lines are parallel. [3]

SOLUTION :

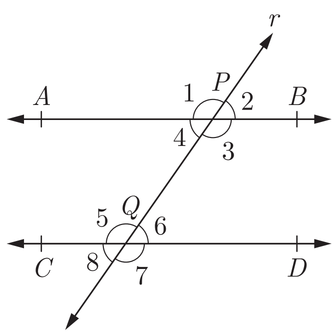
If a transversal intersects two lines, such that a pair of alternate interior angles is equal, then the two lines are parallel. [Converse of theorem]

Given : A transversal r intersects two lines AB and CD at points P and Q, respectively such that

$$\angle 3 = \angle 5$$

and $$\angle 4 = \angle 6$$

To prove : AB is parallel to CD.
i.e., $AB || CD$



Proof : We have,

$$\angle 3 = \angle 5 \quad \text{[Alternate interior angles]} \quad ...(1)$$

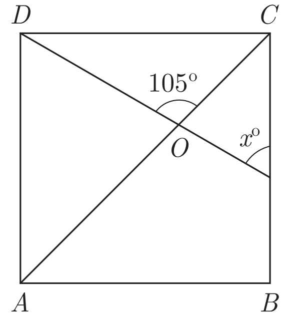
and $$\angle 3 = \angle 1 \quad \text{[Vertically opposite angles]} \quad ...(2)$$

Then, from eqs (1) and (2), we have

$$\angle 1 = \angle 5$$

But these are corresponding angles.
So, $AB || CD$ [By converse of corresponding angles axiom]

29. In the adjacent figure, $ABCD$ is a square. A line segment DX cuts the side BC at X and the diagonal AC at O, such that $\angle COD = 105^{\circ}$. Find the value of x. [3]



SOLUTION :

Given, $ABCD$ is a square and $\angle COD = 105^{\circ}$
We know that, the diagonal AC of square $ABCD$ will bisect the $\angle C$.

$$\therefore \quad \angle OCX = \frac{1}{2}\angle C = \frac{1}{2} \times 90^{\circ} = 45^{\circ} \quad ...(1)$$

Now, we have

$$\angle COD + \angle COX = 180^{\circ} \quad \text{[Linear pair axiom]}$$

$$\Rightarrow \quad 105^{\circ} + \angle COX = 180^{\circ}$$

$$\angle COX = 180^{\circ} - 105^{\circ} = 75^{\circ} \quad ...(2)$$

Now, in ΔCOX,

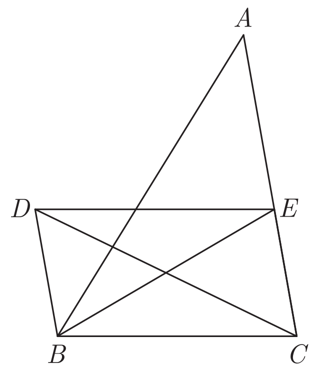
$$\angle COX + \angle OXC + \angle OCX = 180^{\circ}$$

[Angle sum property of a triangle]

$$\Rightarrow \quad 75^{\circ} + x + 45^{\circ} = 180^{\circ}$$

$$x = 180^{\circ} - 120^{\circ} = 60^{\circ}$$

30. In the given figure, $BD || CA$, E is the mid-point of CA and $DB = \frac{1}{2}CA$. Prove that $ar(\Delta ABC) = 2\,ar(\Delta DBC)$. [3]



SOLUTION :

Given : $BD || CA$, E is mid-point of CA, is BE i.e. median of ΔABC, also $BD = \frac{1}{2}CA$.

To prove : $ar(\Delta ABC) = 2\,ar(\Delta DBC)$
Proof : We know that median of a triangle divides it into two triangles of equal area.

$$\therefore \quad ar(\Delta ABE) = ar(\Delta EBC)$$

$$\Rightarrow \quad ar(\Delta EBC) = \frac{1}{2}ar(\Delta ABC) \quad ...(1)$$

Also, $$ar(\Delta DBC) = ar(\Delta EBC) \quad ...(2)$$

[$\because$ Triangles on same base and between same parallel lines are equal in area]

From eqs.(1) and (2), we get

$$ar(\Delta DBC) = \frac{1}{2} ar(\Delta ABC)$$

$$\Rightarrow \quad ar(\Delta ABC) = 2 ar(\Delta DBC)$$

or

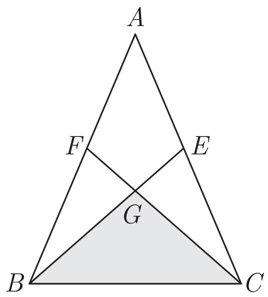
The medians BE and CF of a ΔABC intersect at G. Prove that : $ar(\Delta GBC) = ar(AFGE)$.

SOLUTION :

Given : Medians BE and CF of ΔABC intersect at G.
To prove : $ar(\Delta GBC) = ar(AFGE)$.
Proof : We have

$$ar(\Delta FBC) = \frac{1}{2} ar(\Delta ABC) \quad ...(1)$$

[$\because$ Median CF divides ΔABC into two triangles of equal areas]



Similarly,

$$ar(\Delta EBC) = \frac{1}{2} ar(\Delta ABC) \quad ...(2)$$

From (1) and (2), we get

$$ar(\Delta FBC) = ar(\Delta EBC) \quad ...(3)$$

Subtracting $ar(\Delta BGC)$ from both sides of (3), we get

$$ar(\Delta FBC) - ar(\Delta BGC) = ar(\Delta EBC) - ar(\Delta BGC)$$

$$\Rightarrow \quad ar(\Delta FGB) = ar(\Delta EGC) \quad ...(4)$$

Also, $ar(\Delta ABE) = ar(\Delta BEC)$

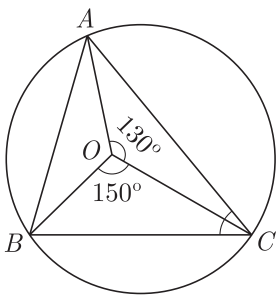
[$\because$ BE is median of ΔABC]

$$ar(\Delta BFG) + ar(AFGE) = ar(\Delta BGC) + ar(\Delta GEC)$$

[Using (4)]

$\Rightarrow \quad ar(AFGE) = ar(\Delta BGC)$ Hence proved.

31. ABC is a triangle inscribed in a circle with centre O. If $\angle AOC = 130^\circ$ and $\angle BOC = 150^\circ$, find $\angle ACB$.[3]



SOLUTION :

Given : A triangle ABC inscribed in a circle with centre O. $\angle AOC = 130^\circ$ and $\angle BOC = 150^\circ$.
To find : $\angle ACB$
Proof : $\angle AOB + \angle BOC + \angle AOC = 360^\circ$

[Sum of angles around a point]

$$\Rightarrow \quad \angle AOB = 360^\circ - 150^\circ - 130^\circ = 80^\circ$$

Now, $\angle AOB = 2\angle ACB$

[Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

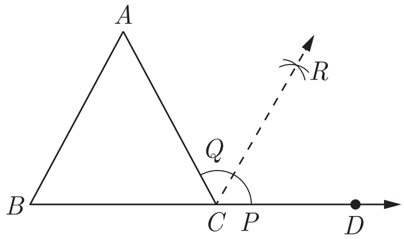
$$\Rightarrow \quad \angle ACB = \frac{1}{2}\angle AOB$$

$$= \frac{1}{2} \times 80^\circ = 40^\circ$$

32. Draw any exterior angle of a triangle and bisect it by using compass only. [3]

SOLUTION :

Steps of Construction :
(i) Draw a ΔABC and produce BC to D to form exterior $\angle ACD$.
(ii) With C as centre and any radius, draw an arc to intersect CD and CA at P and Q, respectively.



(iii) With P and Q as centre and radius more than $\frac{1}{2}PQ$ draw arcs, which intersect each other at point R.
(iv) Join CR, which is the required angle bisector of exterior $\angle ACD$.

33. The dimension of a rectangular box are in the ratio $2:3:4$ and the difference between the cost of covering it with sheet of paper at the rate of ₹ 4 and ₹ 4.50 per sq m is ₹ 416. Find the dimensions of the box. [3]

SOLUTION :

Let the dimensions of the rectangular box be $2x$, $3x$ and $4x$.

$$\therefore \quad \text{Surface area} = 2(2x \times 3x + 3x \times 4x + 4x \times 2x)$$

$$= 2(6x^2 + 12x^2 + 8x^2)$$

$$= 2 \times 26x^2 = 52x^2$$

[$\because$ Surface area $= 2(lb + bh + hl)$]

Cost of covering the box with sheet of paper at the rate of ₹ 4 per m^2

$$= ₹\,(52x^2 \times 4) = ₹\,208x^2$$

and cost of covering the box with sheet of paper at the rate of ₹ 4.50 per m^2

$$= ₹\,(4.50 \times 52x^2) = ₹\,234x^2$$

According to the question,
Difference of the cost at different rate = ₹ 416

$$\Rightarrow \quad 234x^2 - 208x^2 = 416$$

$$26x^2 = 416$$

$$x^2 = 16$$

$$x = 4$$

[On taking the positive square root]

Hence, the dimensions of the box are 8 m, 12 m and 16 m.

or

A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

SOLUTION :

Here, $h = 14\text{ cm}$

Radius of the pencil $(R) = \frac{7}{2}\text{ mm} = 0.35\text{ cm}$

Radius of the graphite $(r) = \frac{1}{2}\text{ mm} = 0.05\text{ cm}$

Volume of the graphite $= \pi r^2 h$

$$= \frac{22}{7} \times 0.05 \times 0.05 \times 14\text{ cm}^3$$

$$= 0.11\text{ cm}^3$$

Volume of the wood $= \pi(R^2 - r^2)h$

$$= \frac{22}{7} \times [(0.35)^2 - (0.05)^2] \times 14\text{ cm}^3$$

$$= \frac{22}{7} \times 0.4 \times 0.3 \times 14\text{ cm}^3$$

$$= 5.28\text{ cm}^3$$

Hence, volume of the wood $= 5.28\text{ cm}^3$ and volume of the graphite $= 0.11\text{ cm}^3$.

34. Cards marked with the numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is a number which is a perfect square. The quick brown fox jumps voer a little lazy dog. [3]

SOLUTION :

The numbers from 2 to 101 which are perfect square are : 4, 9, 16, 25, 36, 49, 64, 81, 100 i.e., squares of 2, 3, 4, 5, 6, 7, 8, 9 and 10 respectively.

Total number of cards having a number which is a perfect square = 9

$\therefore$ p(getting a number which is a perfect square) $= \frac{9}{100}$

Hence, the probability that the number marked on the card which is a perfect square is $\frac{9}{100}$.

Section D

35. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form of $\frac{p}{q}$, where p and q are integers and $p \neq 0$. [4]

SOLUTION :

Let $x = 0.\overline{7}$

$\Rightarrow$ $x = 0.777...$ ...(1)

Multiplying equation(1) by 10

$10x = 7.777...$ ...(2)

Subtracting equation (1) from equation (2), we get

$$10x - x = 7.777 - 0.777$$

$$9x = 7$$

$\Rightarrow$ $x = \frac{7}{9}$

Hence, $0.\overline{7} = \frac{7}{9}$

Again

Let $x = 0.4\overline{7} = 0.4777...$ ...(3)

Here, one digit is not repeating so multiply eq(3) by 10, we get

$10x = 4.777...$ ...(4)

Now, only one digit is repeating, so multiply eq(4) by 10, we get

$$100x = 47.777$$

On subtracting eq(3) from eq(4), we get

$$(100x - 10x) = (47.777) - (4.777)$$

$\Rightarrow$ $90x = 43$

$$x = \frac{43}{90}$$

Hence, $0.4\overline{7} = \frac{43}{90}$

Now,

$$0.6 + 0.\overline{7} + 0.4\overline{7} = \frac{6}{10} + \frac{7}{9} + \frac{43}{90}$$

$$= \frac{54 + 70 + 43}{90} = \frac{167}{90}$$

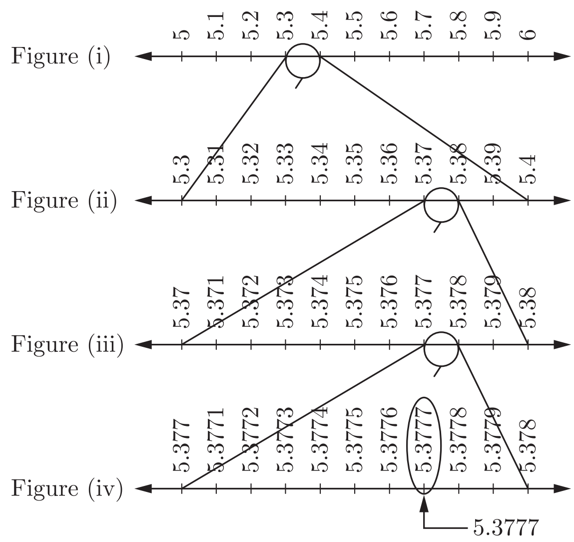
or

Visualise the representation of $5.3\overline{7}$ using successive magnification upto 4 decimal places, that is up to 5.377.

SOLUTION :

Let $x = 5.3\overline{7} = 5.3777$

[up to 4 decimal places]



36. Find the square root of $(x^2 + 4x + 4)(x^2 + 6x + 9)$. [4]

SOLUTION :

We have

$(x^2 + 4x + 4)(x^2 + 6x + 9)$

$$= [x^2 + (2+2)x + 4][x^2 + (3+3)x + 9]$$

[by splitting the middle term]

$$= [x^2 + 2x + 2x + 4][x^2 + 3x + 3x + 9]$$

$$= [x(x+2) + 2(x+2)][x(x+3) + 3(x+3)]$$

$= [(x+2)(x+2)][(x+3)(x+3)]$

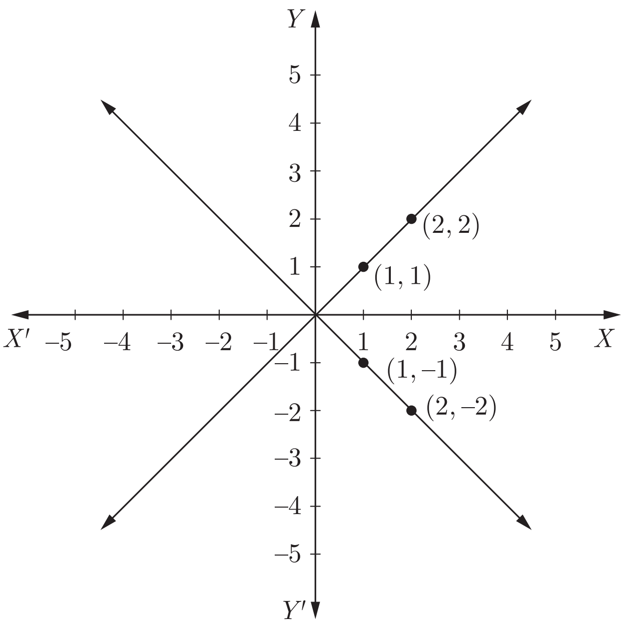
$= (x+2)^2 (x+3)^2$

On taking square root both sides, we get

$$\sqrt{(x^2+4x+4)(x^2+6x+9)} = \sqrt{(x+2)^2(x+3)^2}$$
$$= (x+2)(x+3)$$
$$= x^2+5x+6$$

37. Draw the graph of linear equations $y = x$ and $y = -x$ on the same cartesian plane. Write your observation. [4]

SOLUTION :



Given equation is $y = x$.

When $x = 0$, then $y = 0$

When $x = 1$, then $y = 1$

When $x = 2$, then $y = 2$

Table for $y = x$ is

x	0	1	2
y	0	1	2

and second equation is $y = -x$.

When $x = 0$, then $y = 0$

When $x = 1$, then $y = -1$

When $x = 2$, then $y = -2$

Table for $y = -x$ is

x	0	1	2
y	0	–1	–2

Clearly, both the lines intersect each other at origin.

38. O is the centre of the ΔABC and D is the mid-point of the base BC. Prove that $\angle BOD = \angle A$. [4]

SOLUTION :

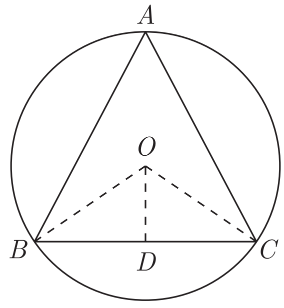
Given : ABC is a triangle and a circle is circumscribed on it having centre O.

Also, D is the mid-point of BC.

To prove : $\angle BOD = \angle A$

or $\angle BOD = \angle BAC$

Construction : Join OB, OD and OC.



Proof : In ΔODB and ΔODC,

$OB = OC$ [Radii of same cirlce]

$BD = DC$ [Since, D is mid-point of BC]

$OD = OD$ [Common sides]

$\therefore \quad \Delta ODB = \Delta ODC$ [By SSS congruence rule]

Then, $\angle BOD = \angle COD$ [By CPCT]

$\therefore \quad \angle BOC = 2\angle BOD$ ...(1)

Also, $\angle BAC = \frac{1}{2}\angle BOC$

[Since, the angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle]

$\therefore \quad \angle BAC = \frac{2}{2}\angle BOD$

[From eq.(1), $\angle BOC = 2\angle BOD$]

$\Rightarrow \quad \angle BAC = \angle BOD$ Hence proved

39. Volume of a right circular cone is $\left(\frac{2200}{7}\right)\text{cm}^3$ and its diameter is 10 cm. Find its curved surface area. (Take $\pi = \frac{22}{7}$) [4]

SOLUTION :

Given, diameter of the base $(d) = 10$ cm

$\therefore$ Radius of the base $(r) = 5$ cm

Let h be the height of the cone.

Also, given, volume of a right circular cone $= \frac{2200}{7}\text{cm}^3$

$$\therefore \quad \frac{1}{3}\pi r^2 h = \frac{2200}{7}$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times h = \frac{2200}{7}$$

$$h = 12 \text{ cm}$$

We have, $h = 12$ cm

and $r = 5$ cm

$$\Rightarrow \quad l = \sqrt{h^2+r^2} = \sqrt{(12)^2+(5)^2}$$
$$= \sqrt{144+25} = \sqrt{169} = 13 \text{ cm}$$

Thus, curved surface area $= \pi r l = \frac{22}{7} \times 5 \times 13$

$$= 204.29 \text{ cm}^2$$

or

Solid sphere of diameter 4 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 12 cm and the water rises by 24 cm, find the number of solid spheres dropped in the water.

SOLUTION :

Diameter of solid sphere $= 4\,\text{cm}$

$\therefore$ Radius of solid spheres $= \frac{4}{2} = 2\,\text{cm}$

Let number of spheres dropped in water be n.
Volume of n spheres $= n\left[\frac{4}{3}\pi(2)^3\right]$

Diameter of cylindrical beaker $= 12\,\text{cm}$

$\therefore$ Radius of cylindrical beaker $= \frac{12}{2} = 6\,\text{cm}$

Height upto which the water level rises $= 24\,\text{cm}$

Volume of water rises $= \pi(6)^2 \times 24\,\text{cm}^3$
As per question,

$$n\left[\frac{4}{3}\pi(2)^3\right] = \pi \times 36 \times 24$$

$$n = \frac{36 \times 24 \times 3}{4 \times 8} = 81$$

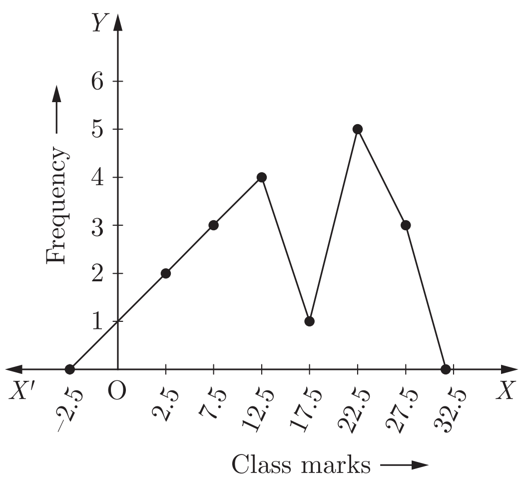
$\therefore$ 81 spheres dropped in the water.

40. Make a frequency polygon for given frequency table. [4]

Class-Interval	Frequency
0 - 5	2
5 - 10	3
10 - 15	4
15 - 20	1
20 - 25	5
25 - 30	3

SOLUTION :

Class-Interval	Class marks	Frequency
0 - 5	2.5	2
5 - 10	7.5	3
10 - 15	12.5	4
15 - 20	17.5	1
20 - 25	22.5	5
25 - 30	27.5	3



Which is the required frequency polygon.