CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-7

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. If $25^{x-1}=5^{2x-1}-100$, then the value of x is. [1]

(a) 3 (b) 2
(c) 4 (d) 1

Ans : (b) 2

$$25^{x-1} = 5^{2x-1}-100 \quad \text{(given)}$$
or, $$5^{2(x-1)} = 5^{2x-1}-100$$
or, $$5^{2x-1}-5^{2x-2} = 100$$

Only $x=2$, satisfy above equation.

2. One of the dimensions of the cuboid whose volume is $16x^2-26x+10$ is [1]

(a) 2 (b) $(8x-5)$
(c) $(x-1)$ (d) All of these

Ans : (d) All of these

$$\begin{aligned}16x^2-26x+10 &= 2(8x^2-13x+5)\\ &= 2(8x^2-8x-5x+5)\\ &= 2(8x-5)(x-1)\end{aligned}$$

3. Point $(0,-2)$ lies [1]

(a) on the x-axis
(b) in the second quadrant
(c) on the y-axis
(d) none of these

Ans : (c) on the y-axis

Since x-coordinate of $(0,-2)$ is zero.

$(0,-2)$ lies on the y-axis.

4. An ordered pair that satisfy an equation in two variables is called its. [1]

(a) Zero (b) Root
(c) Solution (d) Both (b) and (c)

Ans : (c) Solution

5. Priya and Pooja have the same amount of money. If each gets ₹4000 more, how will their new amounts be compared? [1]

(a) Amount with Priya is less than that with Pooja
(b) Amount with Pooja is less than that with Priya
(c) Both have same amount of money
(d) None of these

Ans : (c) Both have same amount of money

Let ₹ x be the money each of Priya and Pooja had.
Priya and Pooja will have amount ₹ $(x+4000)$ each after adding ₹ 4000.
According to Euclid's second axiom, when equals are added to equals, the wholes are equal.
So, Priya and Pooja again have equal amount of money.

6. Which one of the following statements is not false? [1]

(a) If two angles form a linear pair, then each of these angles is of measure 90°
(b) Angles forming a linear pair can both be acute angles.
(c) Both of the angles forming a linear pair can be obtuse angles.
(d) Bisectors of the adjacent angles forming a linear pair form a right angle.

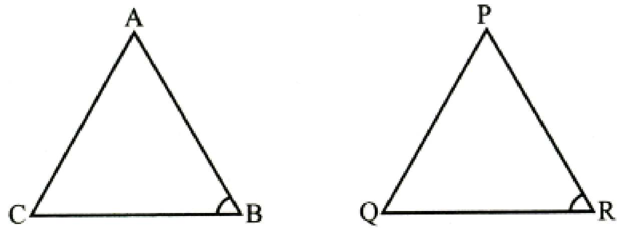
Ans : (d) Bisectors of the adjacent angles forming a linear pair form a right angle.

7. In triangles ABC and PQR, $AB=PQ$ and $\angle B=\angle Q$. The two triangles will be congruent by SAS axiom if [1]

(a) $BC=QR$ (b) $AC=PR$
(c) $AB=QR$ (d) None of these

Ans : (a) $BC=QR$

Given, $AB = PQ$
$\angle B = \angle Q$



In order for ΔABC to be congruent to ΔPQR by SAS axiom corresponding sides BC and QR should be equal.

8. A quadrilateral has three acute angles each measuring 70°. The measure of fourth angle is [1]

(a) 140° (b) 150°

(c) 105° (d) 120°

Ans : (b) 150°

Sum of all angles of a quadrilateral $= 360^\circ$
Fourth angle $= 360^\circ - 210^\circ = 150^\circ$

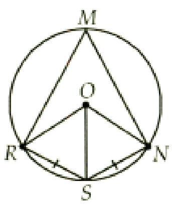
9. Parallelograms on the same base and between the same parallels are equal in [1]

(a) perimeter (b) volume

(c) area (d) weight

Ans : (c) area

10. In the given figure, chord $RS =$ chord NS. How $\widehat{RS}$ is related with $\widehat{NS}$? [1]



(a) $\widehat{RS}$ is smaller than $\widehat{NS}$ (b) Both are equal

(c) $\widehat{RS}$ is greater than $\widehat{NS}$ (d) None of these

Ans : (b) Both are equal

When chords are equal, their arcs are also equal.

(Q.11-Q.15) Fill in the blanks :

11. The construction of a $\triangle DEF$ in which $DE = 7$ cm, $\angle D = 75^\circ$ is possible when $(DE - EF)$ is equal to .... cm. [1]

Ans : 6.5 cm

We know that in a triangle, the difference of two sides is never greater than any side.
i.e., $EF - DF < DE$ i.e., 7 cm
$EF + DF$ will be 6.5 cm.

12. A triangle and parallelogram have the same base and the same area. If the sides of the triangle are 34 cm, 42 cm and 20 cm, then the height of parallelogram having base 42 cm, is equal to .... cm. [1]

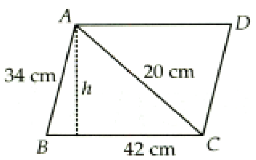
Ans : 8 cm

$$s = \frac{42 + 34 + 20}{2} = \frac{96}{2} = 48 \text{ cm}$$

Area of the triangle

$$= \sqrt{48(48-42)(48-34)(48-20)}$$
$$= \sqrt{48 \times 6 \times 14 \times 28} = 336 \text{ cm}^2$$

$$\text{base} \times \text{height} = 336$$
$$42 \times h = 336$$
$$h = 8 \text{ cm}$$



or

Area of a triangle with the length of sides a, b, c is given by $\sqrt{s(s-a)(s-b)(s-c)}$ by .... formula.

Ans : Heron's

13. A right circular cone is generated by revolving a right angled triangle about one of the sides containing the .... [1]

Ans : right angle

14. The .... of all bars in histogram should be uniform. [1]

Ans : width

15. A .... is an action which results in one of several outcomes. [1]

Ans : Trial

(Q.16-Q.20) Answer the following :

16. The diagonal of a cube is $4\sqrt{3}$ cm. Find its volume. [1]

SOLUTION :

We have, $a\sqrt{3} = 4\sqrt{3}$

$$a = 4$$
$$\text{volume of cube} = a^3 = (4)^3$$
$$= {}^3 = 64 \text{ cm}^3$$

17. In an experiment a coin is tossed 200 times. If the head turns up 120 times, then find the experimental probability of getting a head. [1]

SOLUTION :

$$p \text{ (a head)} = \frac{\text{No. of heads}}{\text{total no. of tosses}}$$
$$= \frac{120}{200} = \frac{3}{5}$$

18. Find a rational number between 8 and 9. [1]

SOLUTION :

Here $9 > 8$
We know that, if x and y are two numbers such that $y > x$.
Then $\frac{x+y}{2}$ is a rational number between x and y.

So, a rational number between 8 and 9 $= \frac{8+9}{2} = \frac{17}{2}$

or

Find the value of $(256)^{0.6} \times (256)^{0.09}$

SOLUTION :

We have, $(256)^{0.16} \times (256)^{0.09} = (256)^{0.16+0.09}$

$$(256)^{0.25} = (256)^{\frac{25}{100}}$$
$$= (256)^{\frac{1}{4}}$$
$$= \left[(4)^4\right]^{\frac{1}{4}}$$
$$= 4$$

19. Find the distance of point $S(-3, 6)$ from y-axis. [1]

SOLUTION :

The distance of a point $S(-3, 6)$ from y-axis is equal to x-coordinate of S. i.e. 3 units.

20. In a grouped frequency distribution, the class intervals are 1-10, 11-20, 21-30, ... Find the class width. [1]

SOLUTION :

This is a discontinuous class interval, so both 1 and 10 are included in the interval.
Hence, the class width = 10

Section B

21. If $x = 9 - 4\sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$. [2]

SOLUTION :

We have, $x = 9 - 4\sqrt{5}$

Then, $\frac{1}{x} = \frac{1}{9 - 4\sqrt{5}} = \frac{(9 + 4\sqrt{5})}{(9 - 4\sqrt{5})(9 + 4\sqrt{5})}$

$= \frac{(9 + 4\sqrt{5})}{(9)^2 - (4\sqrt{5})^2}$

$= \frac{(9 + 4\sqrt{5})}{81 - 80} = 9 + 4\sqrt{5}$

Now, $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= (9 - 4\sqrt{5} + 9 + 4\sqrt{5})^2 - 2$

$= (18)^2 - 2 = 324 - 2 = 322$

or

Prove that $\frac{1}{2 + \sqrt{3}} + \frac{2}{\sqrt{5} - \sqrt{3}} + \frac{1}{2 - \sqrt{5}} = 0$

SOLUTION :

We have

$$\text{LHS} = \frac{1}{2 + \sqrt{3}} + \frac{2}{\sqrt{5} - \sqrt{3}} + \frac{1}{2 - \sqrt{5}}$$

$$= \frac{2 - \sqrt{3}}{(2 + \sqrt{3})(2 - \sqrt{3})} + \frac{2(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} + \frac{2 + \sqrt{5}}{(2 - \sqrt{5})(2 + \sqrt{5})}$$

$$= \frac{2 - \sqrt{3}}{4 - 3} + \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3} + \frac{2 + \sqrt{5}}{4 - 5}$$

$$= 2 - \sqrt{3} + \sqrt{5} + \sqrt{3} - 2 - \sqrt{5}$$

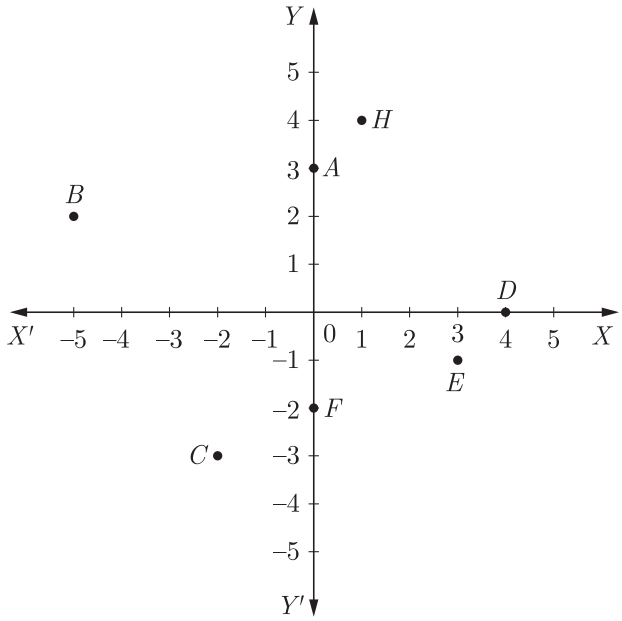
$= 0 = \text{RHS}$ Proved

22. Given the equation of three lines passing through (4, –5). How many more such lines are there and why ? [2]

SOLUTION :

Clearly, the required equations are linear equations in two variables which have solution (4, –5). Three examples of such a linear equation are $x + y = -1$, $x - y = 9$ and $2x + y = 3$. There are infinitely many lines because there are infinitely many linear equations possible which are satisfied by the coordinates of the point (4, –5).

23. From the figure, write the following : [2]

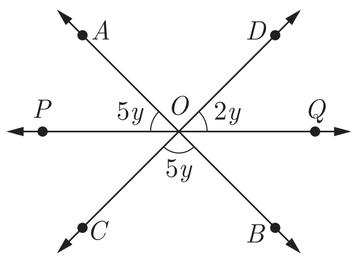


(i) Coordinates of B, C and E
(ii) The point identified by the coordinates (0, –2)
(iii) The abscissa of the point H
(iv) The ordinates of the point D.

SOLUTION :

(i) Coordinates of B, C and E respectively are (–5, 2), (–2, –3) and (3, –1).
(ii) The point identified by the coordinates (0, –2) is F.
(iii) The abscissa of the point H is 1.
(iv) The ordinate of the point D is 0.

24. In the given figure, AB, CD and PQ are three lines concurrent at O. If $\angle AOP = 5y$, $\angle QOD = 2y$ and $\angle BOC = 5y$, then find the value of y. [2]



SOLUTION :

From the given figure, we have

$\angle AOP = \angle BOQ = 5y$ ...(1)
[Vertically opposite angles]

$\angle QOD = \angle COP = 2y$ ...(2)
[Vertically opposite angles]

and $\angle BOC = \angle AOD = 5y$ ...(3)
[Vertically opposite angles]

We know that, sum of angles around a point is $360°$.

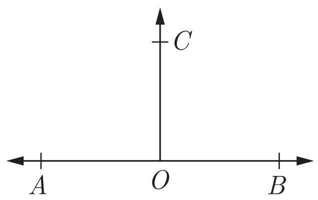
$\therefore \quad \angle AOP + \angle BOQ + \angle QOD + \angle COP$

$$+\angle BOC + \angle AOD = 360^\circ$$
$$5y + 5y + 2y + 2y + 5y + 5y = 360^\circ$$
[From eqs.(1), (2) and (3)]
$$24y = 360^\circ$$
$$\therefore \quad y = \frac{360^\circ}{24} = 15^\circ$$

or

If ray OC stands on line AB such that $\angle AOC = \angle COB$, then show that $\angle AOC = 90^\circ$.

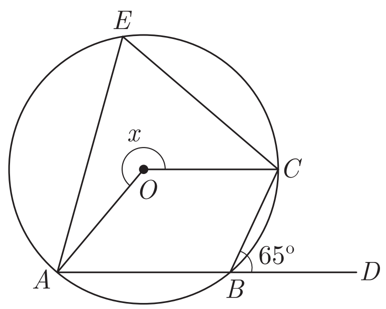
SOLUTION :



Since ray OC stands on line AB. Therefore,

$$\angle AOC + \angle COB = 180^\circ \quad \text{[Linear pair]} \quad ...(1)$$
But $\angle AOC = \angle COB$ [Given]
$$\therefore \angle AOC + \angle AOC = 180^\circ$$
$$2\angle AOC = 180^\circ$$
$$\Rightarrow \quad \angle AOC = 90^\circ$$

25. If O is the centre of the circle, then find the value of x in the given figure. [2]



SOLUTION :

We have,

$$\angle ABC + \angle CBD = 180^\circ \quad \text{[Linear pair axiom]}$$
$$\angle ABC + 65^\circ = 180^\circ \quad [\angle CBD = 65^\circ, \text{given}]$$
$$\Rightarrow \quad \angle ABC = 180^\circ - 65^\circ = 115^\circ$$
Now, we know that
$$\text{Reflex } \angle AOC = 2\angle ABC$$
$$\Rightarrow \quad x = 2 \times 115^\circ = 230^\circ$$
Hence, the value of x is 230°.

26. A conical tent is to accommodate 11 persons. Each person must have 4 sq m of the space on the ground and 20 cubic metre of air to breathe. Find the height of the cone. [2]

SOLUTION :

Let h m be the height, r m be the radius of base of the cone. Since, the tent can accommodate 11 persons and each person requires 4 sq m of the space on the ground and 20 cu m of air.

$$\text{Area of the base} = (11 \times 4) = 44 \text{ m}^2$$
$$\Rightarrow \quad \pi r^2 = 44 \text{ m}^2 \quad ...(1)$$
$[\because \text{Area of base} = \pi r^2]$
$$\text{Volume of the cone} = (11 \times 20) = 220 \text{ m}^3$$
$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 220 \text{ m}^3 \quad ...(2)$$
On dividing eq.(2) by eq.(1), we get
$$\frac{\frac{1}{3}\pi r^2 h}{\pi r^2} = \frac{220}{44}$$
$$\frac{h}{3} = 5$$
$$\Rightarrow \quad h = 15 \text{ m}$$
Hence, the height of the cone is 15 m.

or

The diameter of a roller, 120 cm long is 84 cm. It takes 500 complete revolutions to level a playground. Find the cost of levelling it at the rate of ₹ 25 per sq metre.

SOLUTION :

$$\text{Length of roller } (h) = 120 \text{ cm}$$
$$\text{Radius of the roller } (r) = 42 \text{ cm}$$
Curved surface area of the roller
$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 42 \times 120 \text{ cm}^2$$
But, area covered by the roller in 1 revolution
$$= \text{Curved surface area of the roller}$$
$$= 2 \times \frac{22}{7} \times 42 \times 120 \text{ cm}^2$$
$\therefore$ Area covered by the roller in 500 revolutions
$$= 2 \times \frac{22}{7} \times 42 \times 120 \times 500 \text{ cm}^2$$
$$= 15840000 \text{ cm}^2 = 1584 \text{ m}^2$$
$\therefore$ Cost of levelling the ground
$$= ₹\,25 \times 1584 = ₹\,39600$$

Section C

27. (i) Without actually calculating the cubes, find the value of $48^3 - 30^3 - 18^3$.

(ii) Without finding the cubes, factorise $(x-y)^3 + (y-z)^3 + (z-x)^3$ [3]

SOLUTION :

We know that $x^3 + y^3 + z^3 - 3xyz$
$$= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$$
If $x + y + z = 0$, then $x^3 + y^3 + z^3 - 3xyz = 0$ or $x^3 + y^3 + z^3 = 3xyz$

(i) We have to find the value of $48^3 - 30^3 - 18^3 = 48^3 + (-30)^3 + (-18)^3$

Here, $48 + (-30) + (-18) = 0$

So, $48^3 + (-30)^3 + (-18)^3 = 3 \times 48 \times (-30) \times (-18)$
$$= 77760$$

(ii) Here, $(x-y)+(y-z)+(z-x) = 0$

Therefore, $(x-y)^3+(y-z)^3+(z-x)^3$

$$= 3(x-y)(y-z)(z-x)$$

or

Find the value of k, if $x+k$ is the factor of the polynomials :

(i) $x^3+kx^2-2x+k+5$

(ii) $x^4-k^2x^2+3x-6k$

SOLUTION :

Using factor theorem, if $x+k$ is a factor of $p(x)$ then $p(-k)=0$. Here for all expressions $x=-k$.

(i) Let $p(x) = x^3+kx^2-2x+k+5$

$\therefore \quad p(-k) = (-k)^3+k(-k)^2-2(-k)+k+5$

$= 0$

$-k^3+k^3+2k+k+5 = 0$

$\Rightarrow \quad k = -\frac{5}{3}$

(ii) Let $p(x) = x^4-k^2x^2+3x-6k$

$\therefore \quad p(-k) = (-k)^4-k^2\times(-k)^2+3(-k)-6k$

$= 0$

$k^4-k^4-3k-6k = 0$

$-9k = 0$

$\Rightarrow \quad k = 0$

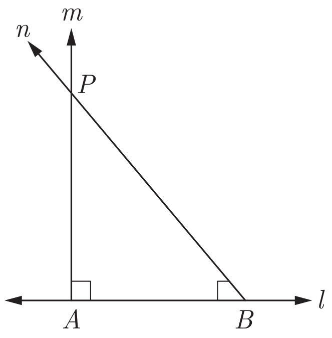
28. Prove that through a given point, we can draw only one perpendicular to a given line. [3]

SOLUTION :

Consider a line l and a point P.

To prove : Only one perpendicular line can be drawn through a given point, i.e., to prove $\angle P = 0°$.

Proof : Suppose there are two intersecting lines passing through the point P which are perpendicular to l.



In ΔAPB, $\angle A+\angle P+\angle B = 180°$ [By angle sum property of a triangle]

$\Rightarrow \quad 90°+\angle P+90° = 180°$

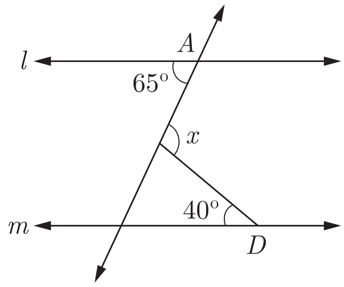
$\angle P = 180°-180°$

$\therefore \quad \angle P = 0°$

So, lines n and m coincide.

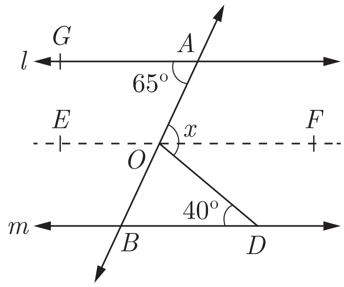
Hence, only one perpendicular line can be drawn through a given point.

29. In the given figure, if $l||m$, then find the value of x. [3]



SOLUTION :

Draw a line EF such that $EF||l||m$.



Then, $\angle FOD = \angle BDO = 40°$

[Alternate interior angles]

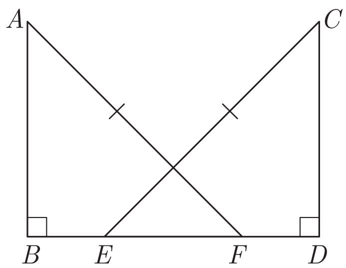
and $\angle FOA = \angle GAO = 65°$

[Alternate interior angles]

Now, $\angle x = \angle FOD+\angle FOA$

$= 40°+65° = 105°$

30. In the given figure, AB and CD are perpendiculars on BD. Also, $AB=CD$ and $AF=CE$. Prove that $BE=FD$. [3]



SOLUTION :

Given : In ΔABF and ΔCDE,

$AB\perp BD$ and $CD\perp BD$ in which $AB=CD$ and $AF=CE$.

To prove : $BE = FD$

Proof : In ΔABF and ΔCDE,

$AB = CD$ [Given]

$AF = CE$ [Given]

and $\angle ABF = \angle CDE = 90°$

$\therefore \quad \Delta ABF \cong \Delta CDE$

[By RHS congruence rule]

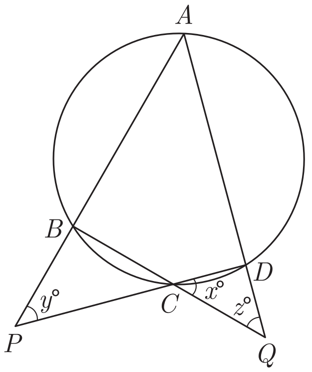
$BF = DE$ [By CPCT]

$BF-EF = ED-EF$

[Subtract both sides from EF]

$\Rightarrow \quad BE = FD$ Hence proved.

31. In the given figure, if $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$, then calculate the values of x, y and z. [3]



SOLUTION :

$$\text{Let} \quad \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k \text{ (say)}$$

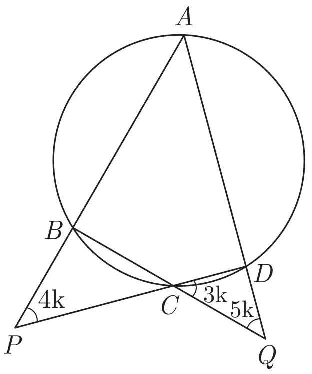
Then, $x = 3k$

$y = 4k$

and $z = 5k$

Since, $\angle DCQ = \angle BCP = 3k$

[Vertically opposite angles]



In ΔDCQ,

$$\angle CDQ = 180^\circ - (3k+5k)$$
$$= 180^\circ - 8k$$

By properties of cyclic quadrilateral,

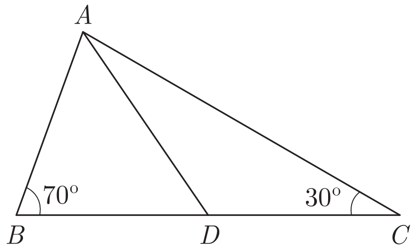
$$\angle ADC + \angle CBA = 180^\circ$$
$$180^\circ - (180^\circ - 8k) + \angle CBA = 180^\circ$$
$$\Rightarrow \quad \angle CBA = 180^\circ - 8k$$
$$\therefore \quad \angle PBC = 180^\circ - \angle ABC$$
$$= 180^\circ - (180^\circ - 8k)$$
$$= 8k$$

In ΔPBC, $\angle P + \angle B + \angle C = 180^\circ$

$$\therefore \quad 4k + 8k + 3k = 180^\circ$$
$$k = \frac{180^\circ}{15}$$
$$= 12^\circ$$

$\therefore$ $x = 36^\circ$, $y = 48^\circ$ and $z = 60^\circ$.

32. In the given figure AD bisects $\angle A$. Then, find the relation between the sides AB, AC and DC. [3]



SOLUTION :

Given, AD is the bisector of $\angle BAC$.

In ΔABC, sum of three angles of a triangle is 180°

$$\therefore \quad \angle A = 180^\circ - (\angle B + \angle C)$$
$$= 180^\circ - (70^\circ + 30^\circ) = 80^\circ$$
$$\Rightarrow \quad \angle BAD = \angle DAC = \frac{80^\circ}{2} = 40^\circ$$

[Since, AD is the bisector of $\angle A$]

In ΔABD, $\angle ADB = 180^\circ - (\angle BAD + \angle ABD)$

[Since, sum of three angles of a triangle is 180°]

$$= 180^\circ - (40^\circ + 70^\circ) = 70^\circ$$

and $\angle DBA = \angle ADB$

$\Rightarrow$ $AD = AB$

[Since, sides opposite to equal angles of a triangle are equal]

In ΔADC, $\angle ADC = 180^\circ - (\angle DAC + \angle ACD)$

[Since, sum of three angles of a triangle is 180°]

$$= 180^\circ - (40^\circ + 30^\circ) = 110^\circ$$

and $\angle DCA < \angle CAD < \angle ADC$

[$\because$ $\angle DCA = 30^\circ$, $\angle CAD = 40^\circ$ and $\angle ADC = 110^\circ$]

$\Rightarrow$ $AD < DC < AC$

But $AB = AD$

Therefore,

$$AB < DC < AC$$

33. A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire. [3]

SOLUTION :

Let the length of wire be h cm.

Given, diameter of sphere = 18 cm

$\therefore$Radius of sphere $r_1 = \frac{18}{2} = 9$ cm and

diameter of wire = 4 mm

$\therefore$ Radius of wire $r_2 = \frac{4}{2} = 2\text{ mm} = \frac{2}{10}$ cm

Volume of sphere = Volume of wire

$$\Rightarrow \quad \left(\frac{4}{3} \times \pi \times 9 \times 9 \times 9\right) = \left(\pi \times \frac{2}{10} \times \frac{2}{10} \times h\right)$$
$$\frac{h}{25} = 972$$
$$h = 972 \times 25$$
$$= 24300 \text{ cm} = 243 \text{ m}$$

[100 cm = 1 m]

Hence, the length of the wire is 243 m.

or

The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find
(i) height of the cone
(ii) slant height of the cone
(iii) curved surface area of the cone

SOLUTION :

(i) Let the height of the cone be h cm.
Radius of the base of the cone (r)

$$= \frac{28}{2} = 14\text{ cm}$$

$$\text{Volume of the cone} = 9856\text{ cm}^3$$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 9856$$

$$\frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856$$

$$h = \frac{9856 \times 7 \times 3}{14 \times 14 \times 22}$$

$$= 48\text{ cm}$$

(ii) Let l cm be the slant height of the cone. Then,

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{14^2 + 48^2}$$
$$= \sqrt{196 + 2304}$$
$$= \sqrt{2500}$$
$$\therefore \quad l = 50\text{ cm}$$

(iii) Curved surface area of cone

$$= \pi r l$$
$$= \frac{22}{7} \times 14 \times 50$$
$$= 2200\text{ cm}^3$$

34. Probability of getting a blue ball is $\frac{2}{3}$, from a bag containing 6 blue and 3 red balls. 12 red balls are added in the bag, then find the probability of getting :
(i) a blue ball
(ii) a red ball [3]

SOLUTION :

Given, number of blue balls in a bag = 6
Number of red balls in a bag = 3
Then, total number of balls in a bag = 6 + 3 = 9
After adding 12 balls,
Total number of balls became = 9 + 12 = 21
Number of red balls = 3 + 12 = 15

(i) P (getting a blue ball) $= \dfrac{\text{Number of blue balls}}{\text{Total number of balls}}$

$$= \frac{6}{21} = \frac{2}{7}$$

Hence, the probability of getting a blue ball is $\frac{2}{7}$.

(ii) P (getting a red ball) $= \dfrac{\text{Number of red balls}}{\text{Total number of balls}}$

$$= \frac{15}{21} = \frac{5}{7}$$

Hence, the probability of getting a red ball is $\frac{5}{7}$.

or

Over the past 200 working days, the number of defective parts produced by a machine is given below :

No. of defective parts	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Days	50	32	22	18	12	12	10	10	10	8	6	6	2	2

Determine the probability that tomorrow's output will have :
(i) no defective part
(ii) not more than 5 defective parts
(iii) more than 13 defective parts ?

SOLUTION :

(i) P (the output will have no defective part)

$$= \frac{\text{no. of days on which the output has no defective part}}{\text{total no. of days}}$$

$$= \frac{50}{200} = \frac{1}{4}$$

(ii) P (the output has not more than 5 defective parts)

$$= \frac{\text{no. of days on which the output has not more than 5 defective parts}}{\text{total no. of days}}$$

$$= \frac{50 + 32 + 22 + 18 + 12 + 12}{200}$$

$$= \frac{146}{200} = \frac{73}{100}$$

(iii) P (the output has more than 13 defective parts)

$$= \frac{\text{no. of days on which the output has more than 13 defective parts}}{\text{total no. of days}}$$

$$= \frac{0}{200} = 0$$

Section D

35. Using factor theorem, factorise $x^3 - 6x^2 + 3x + 10$. [4]

SOLUTION :

Let $\quad p(x) = x^3 - 6x^2 + 3x + 10$

Here, constant term = 10 and coefficient of x^3 is 1.
All possible factors of 10 are ± 1, ± 2, ± 5 and ± 10

At $x = -1$, $\quad p(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$
$= -1 - 6 - 3 + 10 = 0$

$$\begin{array}{r|l} & x^2 - 7x + 10 \\ \hline x+1 & x^3 - 6x^2 + 3x + 10 \\ & x^3 + x^2 \\ \hline & -7x^2 + 3x \\ & -7x^2 - 7x \\ \hline & 10x + 10 \\ & 10x + 10 \\ \hline & 0 \end{array}$$

So, $(x+1)$ is a factor of $p(x)$ on dividing $p(x)$ by $(x+1)$, we get,

$$\text{Quotient} = x^2 - 7x + 10$$

So, $\quad p(x) = (x+1)(x^2 - 7x + 10)$

By splitting the middle term, we get

$$p(x) = (x+1)\{x^2-(5+2)(x)+10\}$$

$$[\because 2+5=7 \text{ and } 2\times 5=10]$$

$$= (x+1)\{x^2-5x-2x+10\}$$

$$= (x+1)\{x(x-5)-2(x-5)\}$$

$$= (x+1)(x-2)(x-5)$$

or

If $ab+bc+ca=0$, find the value of

$$\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}.$$

SOLUTION :

Given : $ab+bc+ca = 0$

$\Rightarrow$ $-ab = bc+ca$

$-bc = ca+ab$

and $-ca = ab+bc$

Therefore, $\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}$

$$= \frac{1}{a^2+ab+ca}+\frac{1}{b^2+ab+bc}+\frac{1}{c^2+bc+ca}$$

$$= \frac{1}{a(a+b+c)}+\frac{1}{b(b+a+c)}+\frac{1}{c(c+b+a)}$$

$$= \frac{1}{(a+b+c)}\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]$$

$$= \frac{1}{(a+b+c)}\times\left(\frac{ab+bc+ca}{abc}\right)$$

$$= \frac{0}{abc(a+b+c)} = 0$$

36. If $2^x = 3^y = 6^z$, prove that $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$ or $z=\frac{xy}{x+y}$. [4]

SOLUTION :

Let $2^x = 3^y = 6^z = k$

$\Rightarrow$ $2^x = k,\ 3^y = k$ and $6^z = k$

$\Rightarrow$ $2 = (k)^{\frac{1}{x}},\ 3 = (k)^{\frac{1}{y}}$ and $6 = (k)^{\frac{1}{z}}$

Now, $6 = (k)^{\frac{1}{z}}$

$2\times 3 = (k)^{\frac{1}{z}}$

$(k)^{\frac{1}{x}}\times(k)^{\frac{1}{y}} = (k)^{\frac{1}{z}}$

$\Rightarrow$ $(k)^{\frac{1}{x}+\frac{1}{y}} = (k)^{\frac{1}{z}}$

On equating the power from both sides, we get

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{z}$$

$\therefore$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

$\Rightarrow$ $\frac{1}{z} = \frac{1}{x}+\frac{1}{y}$

$\frac{1}{z} = \frac{x+y}{xy}$

$z = \frac{xy}{x+y}$ Hence proved.

37. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius : [4]

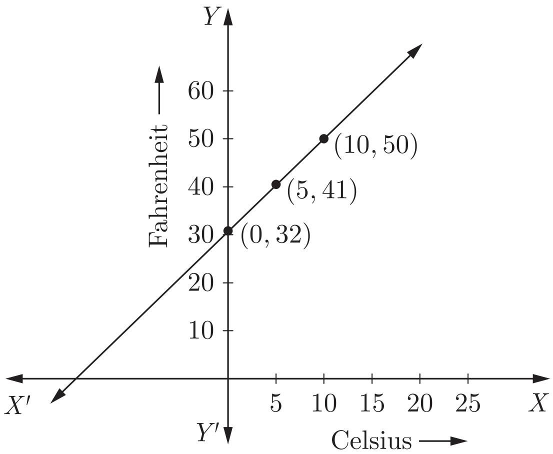
$$F = \left(\frac{9}{5}\right)C+32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.
(ii) If the temperature is $30^\circ C$, what is the temperature in Fahrenheit ?
(iii) If the temperature is $95^\circ F$, what is the temperature in Celsius ?
(iv) If the temperature is $0^\circ C$, what is the temperature in Fahrenheit and if the temperature is $0^\circ F$, what is the temperature in Celsius ?

SOLUTION :

$$F = \left(\frac{9}{5}\right)C+32$$

(i) For, $C=0$, $F=32$
For, $C=5$, $F=41$
For, $C=10$, $F=50$
(ii) For $30^\circ C$, corresponding Fahrenheit temperature is $86^\circ F$.



(iii) For $F=95$,

$$95 = \left(\frac{9}{5}\right)C+32$$

$$\left(\frac{9}{5}\right)C = 95-32 = 63$$

$\Rightarrow$ $C = \frac{5}{9}\times 63 = 35^\circ C$

(iv) For $C = 0^\circ$,

$$F = \left(\frac{9}{5}\right)\times 0+32 = 32^\circ F$$

For $F = 0^\circ$,

$$0 = \left(\frac{9}{5}\right)C+32$$

$$C = \frac{-32\times 5}{9} = \frac{-160}{9} = -17.8^\circ C$$

38. Prove that if any two chords of a circle are drawn, then one which is nearer to the centre, is larger. [4]

SOLUTION :

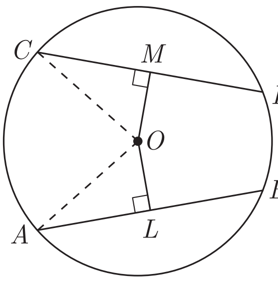
Given : Two chords AB and CD of a circle $C(O, r)$, such that $OL < OM$, where OL and OM are perpendiculars from O on AB and CD, respectively.
To prove : $AB > CD$
Construction : Join OA and OC.

Proof : Since, the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore \quad AL = \frac{1}{2}AB$

and $\quad CM = \frac{1}{2}CD$

In right angled ΔOLA and ΔOMC, we have



$$OA^2 = OL^2 + AL^2$$

and $\quad OC^2 = OM^2 + CM^2$

[By Pythagoras theorem]

$\Rightarrow \quad AL^2 = OA^2 - OL^2 \quad$ ...(1)

and $\quad CM^2 = OC^2 - OM^2 \quad$ ...(2)

Now, $\quad OL < OM$

$$OL^2 < OM^2$$

$\Rightarrow \quad -OL^2 > -OM^2$

[Multiplying by $(-)$ on both sides]

$$OA^2 - OL^2 > OA^2 - OM^2$$

[Adding OA^2 on both sides]

$$OA^2 - OL^2 > OC^2 - OM^2 \quad [\because OA^2 = OC^2]$$

$$AL^2 > CM^2 \quad \text{[From eqs.(1) and (2)]}$$

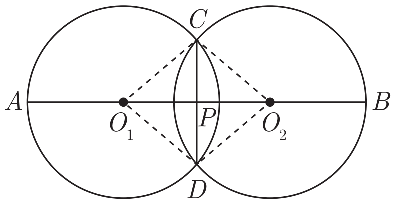
$$AL > CM$$

$$2AL > 2CM \quad \text{[Multiplying by 2]}$$

$\Rightarrow \quad AB > CD \quad$ Hence proved.

or

O_1 and O_2 are the centres of two congruent circles intersecting each other at points C and D. The line joining their centres intersects the circles in points A and B such that $AB > O_1O_2$. If $CD = 6$ cm and $AB = 12$ cm, determine the radius of either circle.



SOLUTION :

Let radius of each circle $= r$ cm

$$AB = 12 \text{ cm}$$

$\therefore \quad O_1O_2 = 12 - 2r$

Now, CD is the common chord of the two circles and O_1O_2 is the line segment that joins the centres.

[Radii of congruent circles]

As we know that line joining the centres of two circles is perpendicular bisector of the common chord.

$\therefore \quad O_1O_2 \perp CD$ and O_1O_2 bisects CD

$\therefore \quad CP = \frac{1}{2} \times CD = 3$ cm

and $\quad O_1P = \frac{1}{2}(O_1O_2) = \frac{1}{2}(12 - 2r)$

$$= (6 - r) \text{ cm}$$

Now in right ΔCPO_1,

$\Rightarrow \quad (O_1C)^2 = (O_1P)^2 + (PC)^2$

$$r^2 = (6 - r)^2 + (3)^2 = 36 + r^2 - 12r + 9$$

$$12r = 45$$

$\Rightarrow \quad r = \frac{45}{12} = 3.75$ cm

39. Find the weight of a lead pipe 3.5 m long, if the external diameter of the pipe is 2.4 cm and the thickness of the lead is 2 mm and 1 cu. cm of lead weights 11 g. [4]

SOLUTION :

We have

R = External radius of the pipe = 1.2 cm

r = Internal radius of the pipe

= External radius − Thickness

$= 1.2 - 0.2 = 1$ cm

h = Length of the pipe

$= 3.5$ m $= 350$ cm

$\therefore$ Volume of lead

= External volume − Internal volume

$$= \pi R^2 h - \pi r^2 h = \pi(R^2 - r^2)h$$

$$= \pi(R + r)(R - r)h$$

$$= \left[\frac{22}{7} \times (1.2 + 1) \times (1.2 - 1) \times 350\right]$$

$$= \frac{22}{7} \times 2.2 \times 0.2 \times 350 = 484 \text{ cm}^3$$

Since, 1 cu cm of lead weights 11 g.

$\therefore$ Weight of the pipe $= (484 \times 11)$g

$= 5324$ g $= 5.324$ kg

40. Prove that $\sum_{i=1}^{n}(x_i - \overline{x}) = 0$, where $\overline{x}$ is the mean of the n observations $x_1, x_2, ... x_n$. [4]

SOLUTION :

We know that

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + ... + x_n}{n}$$

$\Rightarrow \quad n\overline{x} = x_1 + x_2 + ... + x_n \quad$ ...(1)

Now,

$$\sum_{i=1}^{n}(x_i - \overline{x}) = [(x_1 - \overline{x}) + (x_2 - \overline{x}) + ...(x_n - \overline{x})]$$

$$= (x_1 + x_2 + ...x_n) - (\overline{x} + \overline{x} + ...\overline{x})$$

[n times]

$= n\overline{x} - n\overline{x} \quad$ [using (1)]

$= 0$