CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-6

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The rational number between 1/2 and 1/3 is [1]

(a) 2/5 (b) 1/5

(c) 3/5 (d) 4/5

Ans : (a) 2/5

Since, $\frac{1}{2} = 0.5$,

$\frac{1}{3} = 0.\bar{3}$

Rational number lies between $0.\bar{3}$ and 0.5 is

$$0.4 = \frac{4}{10} = \frac{2}{5}$$

2. Which of the following algebraic expressions is not a polynomial ? [1]

(a) $\frac{17}{2}x^2 + x - 3$ (b) $\sqrt{7}\,x^3 + 3x^{2/3} - 8$

(c) 3 (d) 0

Ans : (b) $\sqrt{7}\,x^3 + 3x^{2/3} - 8$

The expression $\sqrt{7}\,x^3 + 3x^{2/3} - 8$ is not a polynomial as second term has fractional exponent $= 2/3$

3. Point $(-2, 3)$ lies in the [1]

(a) first quadrant (b) second quadrant

(c) third quadrant (d) fourth quadrant

Ans : (b) second quadrant

Since x-coordinate of $(-2, 3)$ is negative and y -coordinate is positive. Point $(-2, 3)$ lies in the Ii quadrant.

4. The distance between $M(-1, 5)$ and $N(x, 5)$ is 8 units. The value of x is [1]

(a) -9 or 9 (b) -7 or 9

(c) -9 or 7 (d) -7 or -9

Ans : (c) -9 or 7

The point $M(-1, 5)$ and $N(x, 5)$ lie on a line parallel to x-axis because their ordinates are same. Since the distance between the points is 8 units, therefore the value of x is -9 or 7.

5. Euclid's Postulate 1 is [1]

(a) A straight line may be drawn from any point to any other point.

(b) A terminated line can be produced indefinitely.

(c) All right angles are equal to one another.

(d) None of these

Ans : (a) A straight line may be drawn from any point to any other point.

Euclid's Postulate 1 : 'A straight line may be drawn from any point to any other point.'

6. If the supplement of an angle is three times its complement, then angle is [1]

(a) 40° (b) 35°

(c) 50° (d) 45°

Ans : (d) 45°

Let the angle be x.

Complement of $x = 90^\circ - x$

Supplement of $x = 180^\circ - x$

Given that, $180^\circ - x = 3(90^\circ - x)$

$$180^\circ - x = 270^\circ - 3x$$

$$2x = 270^\circ - 180^\circ$$

$$2x = 90^\circ \quad x = 45^\circ$$

7. In triangles ABC and $5RQP$, if $AB = AC, \angle C = \angle P$ and $\angle B = \angle Q$, then two triangles are [1]

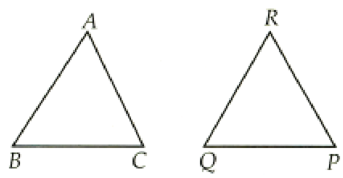
(a) isosceles but not necessarily congruent

(b) isosceles and congruent

(c) congruent but not isosceles

(d) neither congruent nor isosceles

Ans : (a) isosceles but not necessarily congruent



We have $AB = AC$, then $\angle B = \angle C$ ...(1)

$\angle C = \angle P$ [Given] ...(2)

$\angle B = \angle Q$ ...(3)

and $\angle P = \angle Q$ [From (1), (2) and (3)]

$\Rightarrow$ $PR = QR$

$\Rightarrow$ Triangles are isosceles.

8. A quadrilateral having only one pair of opposite sides parallel is called a [1]

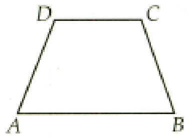
(a) square (b) rhombus

(c) trapezium (d) parallelogram

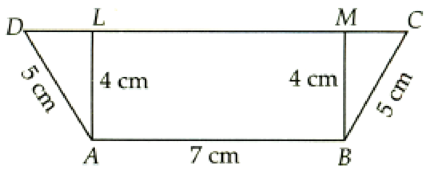
Ans : (c) trapezium

A quadrilateral having exactly one pair of parallel sides is called a trapezium.

In given figure, $ABCD$ is a trapezium in which $AB \parallel DC$.



9. In figure, $ABCD$ is a trapezium in which $AB \parallel DC$. Find the length of DC. [1]



(a) 17 cm (b) 11 cm

(c) 13 cm (d) 15 cm

Ans : (c) 13 cm

$$AL = BM = 4 \text{ cm}$$
$$AD = BC = 5 \text{ cm}$$
$$AB = ML = 7 \text{ cm}$$

In right ΔALD, $DL^2 = AD^2 - AL^2 = 5^2 - 4^2 = 9$

$$DL = 3 \text{ cm}$$

Similarly, $CM = 3 \text{ cm}$

Now, $CD = CM + ML + DL$

$$= 3 + 7 + 3 = 13 \text{ cm}$$

10. In a cyclic quadrilateral $ABCD$, if two sides are parallel, which of the following statements is definitely false? [1]

(a) Remaining two sides are equal

(b) Diagonals are not equal

(c) Diagonals intersect at the centre of circle

(d) Both (a) and (c)

Ans : (b) Diagonals are not equal

(Q.11-Q.15) Fill in the blanks :

11. The construction of a ΔLMN in which $LM = 8$ cm, $\angle L = 45°$ is possible when $(MN + LN)$ is .... cm.[1]

Ans : 9 cm

We know that sum of two sides of a triangle is always greater than third side.

$MN + LN > LM$ i.e., 8 cm

$MN + LN$ will be 9 cm

12. The lengths of the three sides of a triangular field are 40 m, 24 m and 32 m respectively. The area of the triangle is .... m^2. [1]

Ans : 384 m^2

Here, $a = 40$ m

$b = 24$ m

and $c = 32$ m

$$s = \frac{1}{2}(40 + 24 + 32) = 48 \text{ m}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{48 \times 8 \times 24 \times 16} = 348 \text{ m}^2$$

13. Cube is a special form of .... [1]

Ans : Cuboid

14. The .... of a class interval is called its class mark. [1]

Ans : mid-point

15. Probability of an event can be any .... from 0 to 1. [1]

Ans : Fraction

(Q.16-Q.20) Answer the following :

16. Determine the degree of the polynomial : $x^3(2 - x^3)$. [1]

SOLUTION :

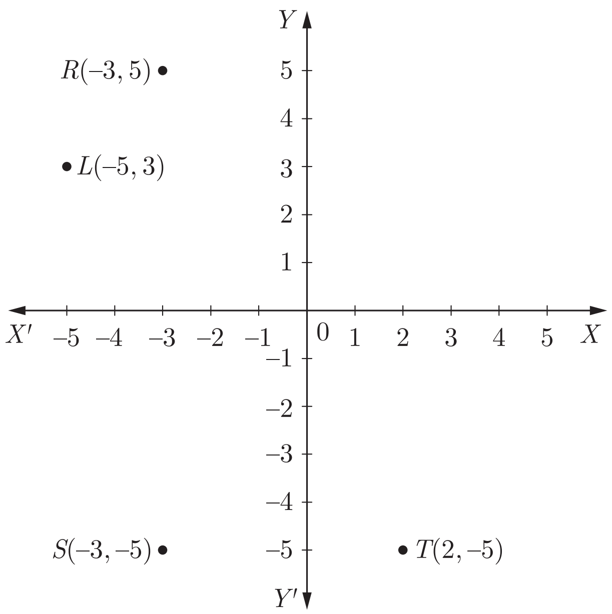
We have, $x^3(2 - x^3) = 2x^3 - x^6$

Here, the highest power of x is 6, so its degree is 6

17. In the given figure, find the point identify the coordinate $(-5, 3)$. [1]

SOLUTION :

For the point $(-5, 3)$, X-coordinate is negative and Y-coordinate is positive. So, it will lie in II quadrant. Also, its perpendicular distance from Y-axis is 5 units and from X-axis is 3 units. So, the required point is L.



18. Solve the equation $m - 25 = 40$ and state which axiom will use here? [1]

SOLUTION :

Given equation is $m - 25 = 40$

$$m - 25 + 25 = 40 + 25$$
$$m = 65$$

Hence, Euclid's second axiom is used in this question.

19. If the length of a median of an equilateral triangle is x cm, find its area. [1]

SOLUTION :

Let side of an equilateral triangle be a cm.

Then, median of an equilateral triangle $= \frac{\sqrt{3}}{2}a$

$$x = \frac{\sqrt{3}}{2}a$$
$$a = \frac{2x}{\sqrt{3}} \text{ cm}$$

Now, area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times a \times x = \frac{1}{2} \times \frac{2x}{\sqrt{3}} \times x$$
$$= \frac{x^2}{\sqrt{3}}$$

Hence, the area of triangle is $\frac{x^2}{\sqrt{3}}$ cm^2

20. A boy says that the median of 4, 15, 19, 21 and 6 is 19. What does not the boy understand about finding the median? [1]

SOLUTION :

The boy does not understand that data has to be arranged in ascending or descending order before finding the median.

Section B

21. Simplify : $\frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{2}(\sqrt{5}-\sqrt{3})$ [2]

SOLUTION :

$$\frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{2}(\sqrt{5}-\sqrt{3})$$
$$= \frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + \frac{1}{2}(\sqrt{5}-\sqrt{3})$$
$$= \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{1}{2}(\sqrt{5}-\sqrt{3})$$
$$= \sqrt{5}-\sqrt{3}$$

or

If $\sqrt{3} = 1.732$ and $\sqrt{2} = 1.414$, then find the value of $\frac{1}{\sqrt{3}-\sqrt{2}}$.

SOLUTION :

$$\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$
$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$
$$= \sqrt{3}+\sqrt{2}$$
$$= 1.732 + 1.414 = 3.146$$

22. Write two solutions of the linear equation $x + 2y = 1$. [2]

SOLUTION :

The given equation $x + 2y = 1$ can be written as

$$2y = 1 - x$$
$$y = \frac{1-x}{2}$$

When $x = 1$, then $y = \frac{1-1}{2} = \frac{0}{2} = 0$

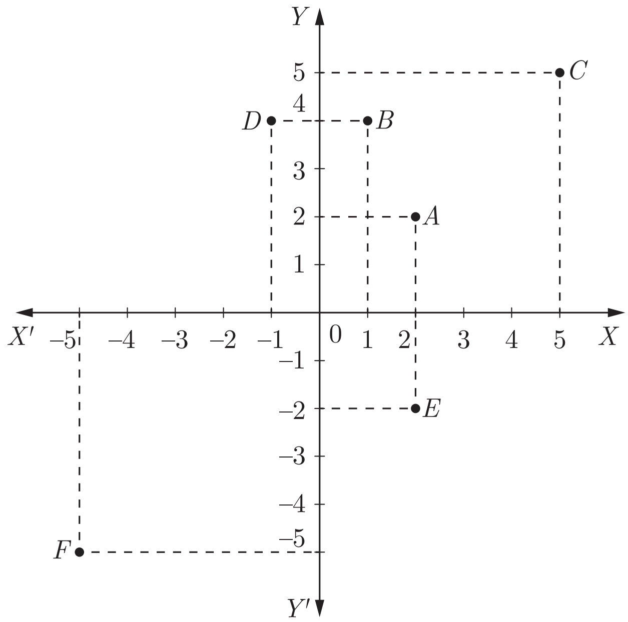
Thus, $x = 1$ and $y = 0$ is a solution of $x + 2y = 1$

When $x = 3$, then $y = \frac{1-3}{2} = -\frac{2}{2} = -1$

Thus, $x = 3$ and $y = -1$ is another solution of $x + 2y = 1$

Hence, the two solutions of $x + 2y = 1$ out of infinitely many solutions are (1, 0) and (3, –1).

23. From the following figure, find the coordinates of the points A, B, C, D, E and F. Which of the points are mirror images in (i) x-axis, (ii) y-axis. [2]



SOLUTION :

The coordinates of A, B, C, D, E and F respectively are : (2, 2), (1, 4), (5, 5), (–1, 4), (2, –2) and (–5, –5).

(i) We see that points A and E are equidistant from the x-axis. So, point A is mirror image of B or point B is mirror image of A.

(ii) No points are there which are mirror images on y -axis.

24. In ΔABC, if $\angle A = (2x - 5^\circ)$, $\angle B = (5x + 5^\circ)$ and $\angle C = (3x + 50^\circ)$, then find the values of x and $\angle C$. [2]

SOLUTION :

Given, $\angle A = (2x - 5^\circ)$,

$\angle B = (5x + 5^\circ)$,

and $\angle C = (3x + 50^\circ)$

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

[By angle sum property of a triangle]

$$\Rightarrow \quad (2x - 5^\circ) + (5x + 5^\circ) + (3x + 50^\circ) = 180^\circ$$

$$10x + 50° = 180°$$
$$10x = 180° - 50°$$
$$10x = 130°$$
$$x = 13°$$

Hence, $\angle C = 3x + 50°$

$$= 3 \times 13° + 50° = 89°$$

or

In ΔABC, if $\angle A : \angle B : \angle C = \frac{1}{2} : \frac{1}{3} : \frac{1}{6}$, then calculate the measures of $\angle A$, $\angle B$ and $\angle C$.

SOLUTION :

Given, $\angle A : \angle B : \angle C = \frac{1}{2} : \frac{1}{3} : \frac{1}{6}$

Then, let $\angle A = \frac{x}{2}$, $\angle B = \frac{x}{3}$ and $\angle C = \frac{x}{6}$

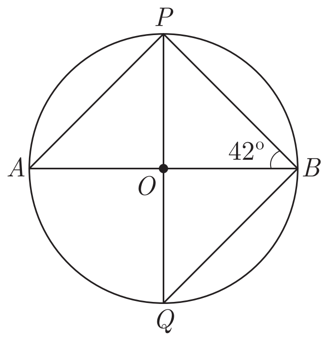
In ΔABC, we know that,

$$\angle A + \angle B + \angle C = 180°$$
$$\frac{x}{2} + \frac{x}{3} + \frac{x}{6} = 180°$$
$$\frac{3x + 2x + x}{6} = 180°$$
$$6x = 180° \times 6$$
$$x = 180°$$

$\therefore$ $\angle A = \frac{x}{2} = 90°$

$$\angle B = \frac{x}{3} = 60°$$
$$\angle C = \frac{x}{6} = 30°$$

25. In the following figure, find the measure of $\angle PQB$, where O is the centre of the circle. [2]



SOLUTION :

We know that, angle in a semi-circle is a right angle.

$\therefore$ $\angle APB = 90°$

In ΔAPB, we have

$$\angle PAB + \angle PBA + \angle APB = 180°$$
$$\angle PAB + 42° + 90° = 180°$$
$$\angle PAB = 48°$$

Now, $\angle PQB = \angle PAB$

[$\because$ Angles in the same segment are equal]

$\Rightarrow$ $\angle PQB = 48°$

26. The height of a cylinder is 15 cm and the curved surface area is $660\ \text{cm}^2$. Find its radius. [2]

SOLUTION :

Given that, $h = 15$ cm

and curved surface area $= 2\pi rh = 660\ \text{cm}^2$

$\therefore$ $2 \times \frac{22}{7} \times r \times 15 = 660$

$\Rightarrow$ $r = \frac{660 \times 7}{2 \times 22 \times 15} = 7$

$\therefore$ Radius of the cylinder is 7 cm.

or

The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

SOLUTION :

Let diameter of the earth $= 2r$

Then, radius of the earth $= r$

$\therefore$ Diameter of the moon $= \frac{2r}{4} = \frac{r}{2}$

$\therefore$ Radius of the moon $= \frac{r}{4}$

Now, surface area of the moon

$$= 4\pi\left(\frac{r}{4}\right)^2 = \frac{\pi r^2}{4} \quad ...(1)$$

Surface area of the earth $= 4\pi r^2$ ...(2)

$\therefore$ Required ratio $= \frac{\frac{\pi r^2}{4}}{4\pi r^2} = \frac{\pi r^2}{4 \times 4\pi r^2}$

$$= \frac{1}{16} = 1 : 16$$

Section C

27. If $(x + 4)$ is a factor of the polynomial $x^3 - x^2 - 14x + 24$, find its other factors. [3]

SOLUTION :

Let $p(x) = x^3 - x^2 - 14x + 24$

Given, $(x + 4)$ is a factor of $p(x)$.

Now, we divide $p(x)$ by $(x + 4)$ by long division method.

$$\begin{array}{r|l} & x^2 - 5x + 6 \\ x + 4 & x^3 - x^2 - 14x + 24 \\ & x^3 + 4x^2 \\ \hline & -5x^2 - 14x \\ & -5x^2 - 20x \\ \hline & 6x + 24 \\ & 6x + 24 \\ \hline & 0 \end{array}$$

Now, factorise the quotient as :

$$x^2 - 5x + 6 = x^2 - 3x - 2x + 6$$
$$= x(x - 3) - 2(x - 3)$$

$$= (x-3)(x-2)$$

Hence, other factors are $(x-2)$ and $(x-3)$.

or

Let R_1 and R_2 are the remainders when polynomial $f(x) = 4x^3 + 3x^2 + 12ax - 5$ and $g(x)\ 2x^3 + ax^2 - 6x - 2$ are divided by $(x-1)$ and $(x-2)$ respectively. If $3R_1 + R_2 - 28 = 0$, find the value of a.

SOLUTION :

Given :

$$f(x) = 4x^3 + 3x^2 + 12ax - 5$$

and $$g(x) = 2x^3 + ax^2 - 6x - 2$$

Now, R_1 = remainder when $f(x)$ is divided by $x-1$

$$\Rightarrow \quad R_1 = f(1)$$

$$R_1 = 4(1)^3 + 3(1)^2 + 12a(1) - 5$$

$$= 4 + 3 + 12a - 5$$

$$= 12a + 2$$

And R_2 = remainder when $g(x)$ is divided by $x-2$

$$\Rightarrow \quad R_2 = g(2)$$

$$= 2(2)^3 + a(2)^2 - 6(2) - 2$$

$$= 16 + 4a - 12 - 2$$

$$= 4a + 2$$

Substituting the value of R_1 and R_2 in $3R_1 + R_2 - 28 = 0$, we get

$$\Rightarrow \quad 3(12a+2) + 4a + 2 - 28 = 0$$

$$36a + 6 + 4a - 26 = 0$$

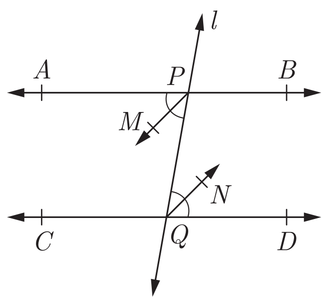
$$40a = 20$$

$$a = \frac{20}{40} = \frac{1}{2}$$

28. If two parallel lines are intersected by a transversal, then prove that the bisectors of two alternate interior angles are parallel. [3]

SOLUTION :

Given : $AB || CD$ and a transversal l intersects them at P and Q respectively.
Also, PM and QN are the bisectors of $\angle APQ$ and $\angle DQP$.



To prove : $PM || QN$

Proof : $$\angle APQ = \angle DQP$$

$$\Rightarrow \quad \frac{1}{2}\angle APQ = \frac{1}{2}\angle DQP$$

[Dividing both sides by 2]

$$\angle MPQ = \angle NQP$$

[Since, PM and NQ are bisectors of $\angle APQ$ and $\angle PQD$]

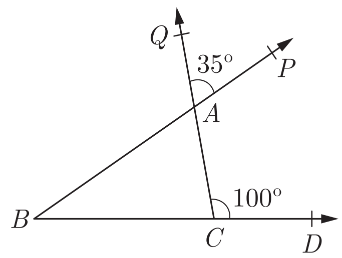
But these are alternate angles.

$$\therefore \quad PM || QN$$

Hence proved.

or

Side BC, CA and BA of triangle ABC produced to D, Q, P respectively as shown in the figure. If $\angle ACD = 100^\circ$ and $\angle QAP = 35^\circ$, find all the angles of a triangle.



SOLUTION :

We have, $$\angle BAC = \angle QAP$$

[Vertically opposite angles]

$$\Rightarrow \quad \angle BAC = 35^\circ$$

[Given, that $\angle QAP = 35^\circ$]

Also, $$\angle ACB + \angle ACD = 180^\circ$$

[Linear pair axiom]

$$\Rightarrow \quad \angle ACB + 100^\circ = 180^\circ$$

$$\angle ACB = 180^\circ - 100^\circ = 80^\circ$$

In ΔABC,

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

[Angle sum property of a triangle]

$$\angle ABC + 80^\circ + 35^\circ = 180^\circ$$

$$\angle ABC + 115^\circ = 180^\circ$$

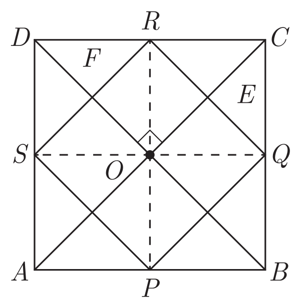
$$\Rightarrow \quad \angle ABC = 180^\circ - 115^\circ = 65^\circ$$

Hence, $\angle ABC = 65^\circ$, $\angle BAC = 35^\circ$ and $\angle ACB = 80^\circ$.

29. The diagonals of a quadrilateral $ABCD$ are perpendicular, show that quadrilateral formed by joining the mid-points of its sides, is rectangle. [3]

SOLUTION :

Given : $ABCD$ is a quadrilateral and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.



To show : Quadrilateral $PQRS$ is a rectangle.

Proof : By mid-point theorem,
In ΔADC, we have S and R are the mid-points of DA and CD respectively.

$\therefore \quad SR \,||\, AC$

and $\quad SR = \frac{1}{2}AC \quad$ ...(1)

In ΔABC, we have P and Q are the mid-points of AB and BC respectively.

$\therefore \quad PQ \,||\, AC$

and $\quad PQ = \frac{1}{2}AC \quad$ ...(2)

From eqs.(1) and (2), we get

$PQ \,||\, SR$

and $\quad PQ = SR = \frac{1}{2}AC$

Since, a pair of opposite sides of a quadrilaterals $PQRS$ is equal and parallel.
So, $PQRS$ is a parallelogram.
Given that, diagonals of a quadrilateral bisect each other at right angles.

$\therefore \quad \angle COD = \angle EOF = 90°$

Now, in ΔBCD, R and Q are the mid-points of CD and BC, respectively.

$RQ \,||\, DB \quad$ [By mid-point theorem]

$\Rightarrow \quad RE \,||\, OF$

Also, $\quad SR \,||\, AC \quad$ [From eq.(1)]

$\Rightarrow \quad FR \,||\, OE$

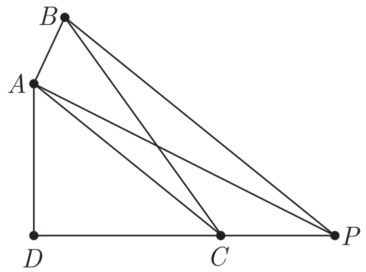
So, $OERF$ is a parallelogram.

$\therefore \quad \angle ERF = \angle EOF = 90°$

[Opposite angles of a parallelogram are equal]

Thus, $PQRS$ is a parallelogram with $\angle R = 90°$.
Hence, $PQRS$ is a rectangle.

30. In the given figure, $ABCD$ is a quadrilateral. BP is drawn parallel to AC and BP meets DC (produced) at P. Prove that $ar(\Delta ADP) = ar$ (quadrilateral $ABCD$). [3]



SOLUTION :

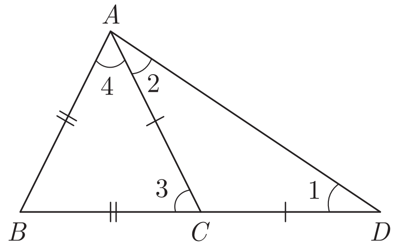
Given : $ABCD$ is a quadrilateral and $BP \,||\, AC$.
To Prove : $ar(\Delta ADP) = ar$ (quadrilateral $ABCD$)
Proof :
Here, ΔACB and ΔACP lie on the same base AC and between the same parallels AC and BP.
So, by the theorem, $ar(\Delta ACB) = ar(\Delta ACP)$
On adding $ar(\Delta ADC)$ both sides, we get

$$ar(\Delta ACB) + ar(\Delta ADC) = ar(\Delta ACP) + ar(\Delta ADC)$$

$\Rightarrow \quad ar(ABCD) = ar(\Delta ADP)$

Hence proved.

31. In the given figure, $AB = BC$ and $AC = CD$. Prove that $\angle BAD : \angle ADB = 3:1$. [3]



SOLUTION :

Given : $AB = BC$ and $AC = CD$
To prove : $\angle BAD : \angle ADB = 3:1$

Proof : $\quad CA = CD \quad$ [Given]

$\Rightarrow \quad \angle 1 = \angle 2 = x \quad$ [Say] ...(1)

[Since, angles opposite to equal sides of a triangle are equal]
Considering ΔADC whose side DC is extended to B, we have

Exterior $\quad \angle 3 = \angle 1 + \angle 2$

[Since, exterior angle is equal to the sum of interior opposite angles]

$\Rightarrow \quad \angle 3 = x + x = 2x \quad$ ...(2)

Also, $\quad BA = BC \quad$ [Given]

$\Rightarrow \quad \angle 3 = \angle 4 \quad$ ...(3)

[Since, angles opposite to equal sides of a triangle are equal]
From eqs. (2) and (3), we get

$\angle 4 = 2x \quad$ ...(4)

Now, $\quad \angle BAD = \angle BAC + \angle CAD$

$= \angle 4 + \angle 2$

$= 2x + x = 3x$

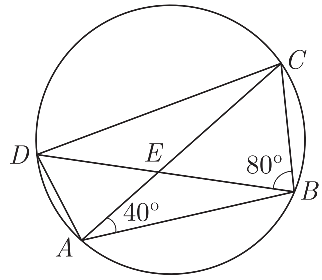
[From eqs.(1) and (4)] ...(5)

Now, $\quad \angle BAD : \angle ADB = 3x : x$

$\therefore \quad \angle BAD : \angle ADB = 3:1$

Hence proved.

32. In the given figure, if $\angle DBC = 80°$ and $\angle BAC = 40°$, then find $\angle BCD$. Further, if $AB = BC$, then find $\angle ECD$. [3]



SOLUTION :

Given, $\quad \angle DBC = 80°$ and $\angle BAC = 40°$

Consider, the chord CD, we find that $\angle CBD$ and $\angle CAD$ are angles in the same segment of the circle.

$\therefore \quad \angle CBD = \angle CAD$

$80° = \angle CAD$

$\Rightarrow \quad \angle CAD = 80^\circ$

Now, $\quad \angle BAD = \angle BAC + \angle CAD$

$\angle BAD = 40^\circ + 80^\circ = 120^\circ \quad ...(1)$

Since, $ABCD$ is a cyclic quadrilateral.

$\therefore \angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \quad 120^\circ + \angle BCD = 180^\circ \quad$ [Using eq.(1)]

$\angle BCD = 60^\circ$

If $AB = BC$, then in ΔABC, we have

$\angle ACB = \angle BAC$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \quad \angle ACB = 40^\circ$

$\therefore \quad \angle ECD = \angle BCD - \angle ACB$

$= 60^\circ - 40^\circ = 20^\circ$

33. The length, breadth and height of a cuboid are 8 m, 6 m and 4 m respectively. Find its total surface area, diagonal and area of four walls. [3]

SOLUTION :

Given,

length of cuboid $(l) = 8\text{ m}$,

breadth $(b) = 6\text{ m}$

and height $(h) = 4\text{ m}$

$\therefore$ Total surface area of cuboid

$= 2(lb + bh + hl)$

$= 2(8 \times 6 + 6 \times 4 + 4 \times 8)$

$= 2(48 + 24 + 32)$

$= 2 \times 104 = 208\text{ m}^2$

Diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{(8)^2 + (6)^2 + (4)^2}$

$= \sqrt{64 + 36 + 16}$

$= \sqrt{116} = 10.77\text{ m}$

Area of the four walls of the cuboid

$= 2(l + b)h$

$= 2(8 + 6) \times 4 = 112\text{ m}^2$

or

A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered with canvas, find the area of the canvas required.

SOLUTION :

Given,

Radius of the conical heap of wheat $(r) = \frac{10.5}{2}\text{m}$

Height of the conical heap of wheat $(h) = 3\text{ m}$

Volume of the conical heap of wheat

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{10.5}{2}\right)^2 \times 3$

$= \frac{173.25}{2} = 86.625\text{ m}^3$

Slant height of the cone l

$= \sqrt{r^2 + h^2}$

$= \sqrt{\left(\frac{10.5}{2}\right)^2 + 3^2}$

$= \sqrt{(5.25)^2 + 3^2}$

$= \sqrt{27.5625 + 9} = \sqrt{36.5625}$

$\Rightarrow \quad l = 6.05$

Area of canvas required = Curved surface area of cone

$= \pi rl$

$= \frac{22}{7} \times \frac{10.5}{2} \times 6.05$

$= 99.825\text{ m}^2$

34. The probabilities of a student getting A, B, C and D grades are 0.35, 0.25, 0.35 and 0.05. Then, find the probability that a student gets atmost grade C. [3]

SOLUTION :

Let, E_1, E_2, E_3 and E_4 denote the events getting grade A, B, C and D respectively.

Then, $\quad P(E_1) = 0.35$,

$P(E_2) = 0.25$,

$P(E_3) = 0.35$

and $\quad P(E_4) = 0.05$

A student who gets atmost grade C can also get grade D.

$\therefore P$ (getting atmost grade C)

$= P$ (getting grade C) $+ P$ (getting grade D)

$= 0.35 + 0.05 = 0.40$

Hence, the probability that a student gets atmost grade C is 0.40.

Section D

35. Find nine rational numbers between 0 and 0.1. [4]

SOLUTION :

Here, $0.1 > 0$, so let $x = 0$, $y = 0.1$ and $n = 9$

Now, $\quad d = \frac{y - x}{n + 1} = \frac{0.1 - 0}{9 + 1}$

$= \frac{0.1}{10} = 0.01$

So, the nine rational numbers between 0 and 0.1 are $(x + d)$, $(x + 2d)$, $(x + 3d)$, $(x + 4d)$, $(x + 5d)$, $(x + 6d)$, $(x + 7d)$, $(x + 8d)$ and $(x + 9d)$

i.e., $(0 + 0.01)$, $(0 + 2 \times 0.01)$, $(0 + 3 \times 0.01)$, $(0 + 4 \times 0.01)$, $(0 + 5 \times 0.01)$, $(0 + 6 \times 0.01)$, $(0 + 7 \times 0.01)$, $(0 + 8 \times 0.01)$ and $(0 + 9 \times 0.01)$

$= 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08$ and 0.09

$= \frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \frac{4}{100}, \frac{5}{100}, \frac{6}{100}, \frac{7}{100}, \frac{8}{100}$ and $\frac{9}{100}$,

$= \frac{1}{100}, \frac{1}{50}, \frac{3}{100}, \frac{1}{25}, \frac{1}{20}, \frac{3}{50}, \frac{7}{100}, \frac{2}{25}$ and $\frac{9}{100}$

36. Factorise, $2x^3 - 5x^2 - 19x + 42$. [4]

SOLUTION :

Let $p(x) = 2x^3 - 5x^2 - 19x + 42$

Here, we see that coefficient of x^3 is not one, so firstly we make the coefficient of x^3 is one.

i.e., $p(x) = 2\left(x^3 - \frac{5}{2}x^2 - \frac{19}{2}x + \frac{42}{2}\right)$

$= 2g(x)$ ...(1)

where, $g(x) = x^3 - \frac{5}{2}x^2 - \frac{19}{2}x + 21$

Here, constant term is 21 and its all factors are ± 1, ± 3, ± 7, ± 21

At $x = 1$,

$$g(1) = (1)^3 - \frac{5}{2}(1)^2 - \frac{19}{2}(1) + 21$$

$$= 1 - \frac{5}{2} - \frac{19}{2} + 21$$

$$= -12 + 22 = 10 \neq 0$$

So, $(x-1)$ is not a factor of $g(x)$.

At $x = 2$,

$$g(2) = (2)^3 - \frac{5}{2}(2)^2 - \frac{19}{2}(2) + 21$$

$$= 8 - \frac{5}{2} \times 4 - \frac{38}{2} + 21$$

$$= 8 + 21 - \frac{20}{2} - \frac{38}{2} = 29 - \frac{58}{2}$$

$$= 29 - 29 = 0$$

So, $(x-2)$ is factor of $g(x)$.

On dividing $g(x)$ by $(x-2)$, we get the quotient $\left(x^2 - \frac{x}{2} - \frac{21}{2}\right)$

$\therefore$ $g(x) = (x-2)\left(x^2 - \frac{x}{2} - \frac{21}{2}\right)$

From eq. (1),

$$p(x) = 2 \times (x-2)\frac{(2x^2 - x - 21)}{2}$$

$$= (x-2)(2x^2 - x - 21)$$

$$= (x-2)[2x^2 - 7x + 6x - 21]$$

[$\because$ $6 \times (-7) = -42$ and $6 - 7 = -1 = b$, by splitting the middle term]

$$= (x-2)[x(2x-7) + 3(2x-7)]$$

$$= (x-2)(2x-7)(x+3)$$

37. The parking charges of a car in a parking lot is ₹ 30 for the first two hours and ₹ 10 per hour for subsequent hours. Taking total parking time to be x hours and total charges as ₹ y, write a linear equation in two variables to express the above statement. Draw a graph for the linear equation and read the charges for five hours. [4]

SOLUTION :

Given, parking charges for the first two hours = ₹ 30 and for subsequent hours = ₹ 10

Total parking time = x hrs and total charges = ₹ y

Then, according to the given condition,

$$30 + 10(x-2) = y$$

$$30 + 10x - 20 = y$$

$$10x + 10 - y = 0$$

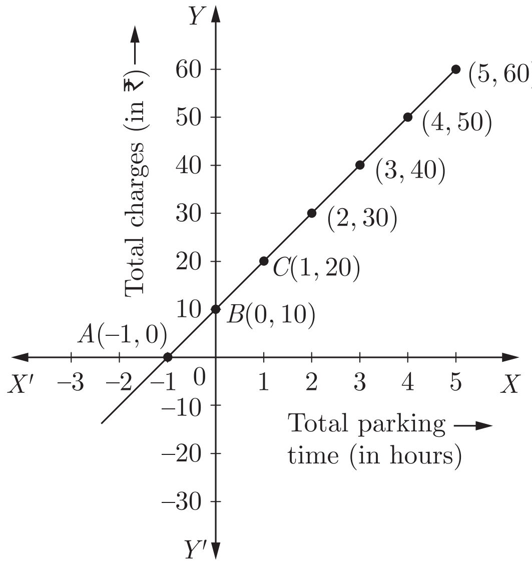
$\Rightarrow$ $$10x - y + 10 = 0$$

which is the required linear equation in two variables.

It can also be written as $y = 10x + 10$ ...(1)

Now, for drawing the graph, we need atleast two solutions of the equation.

When $x = -1$, then $y = -10 + 10 = 0$



When $x = 0$, then $y = 10(0) + 10 = 10$

When $x = 1$, then $y = 10(1) + 10 = 20$

So, we have the following table to draw the graph

x	–1	0	1
y	0	10	20

Here, we have three points $A(-1, 0)$, $B(0, 10)$ and $C(1, 20)$. By plotting these points on the graph paper and joining them, we get a straight line AC, which represents the required graph of linear equation. From the graph, charges for the hours 1, 2, 3, 4 and 5 are ₹ 20, ₹ 30, ₹ 40, ₹ 50 and ₹ 60 respectively.

38. Construct an angle of 150° of the initial point of a given ray and justify the construction. [4]

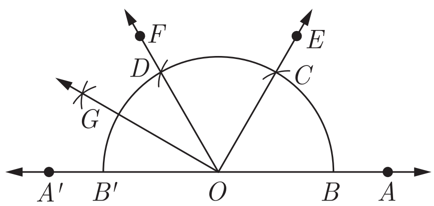
SOLUTION :

Steps of Construction :

(i) First, draw a ray OA with initial point O.

(ii) Produce AO to A' to form ray OA'.

(iii) Taking O as centre and suitable radius, draw an arc of a circle which intersects OA at B and OA' at B'.



(iv) Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at a point C.

(v) Taking C as centre and with the same radius as before, draw an arc intersecting the arc drawn in step (3) at a point D.
(vi) Draw the ray OE passing through C and ray OF passing through D. Then, $\angle EOA = 60°$, $\angle FOE = 60°$ and $\angle FOA' = 60°$.

(vii) Now, taking D and B' as centres and with radius more than $\frac{1}{2}B'D$, draw two arcs to intersect each other at a point say G.
(viii) Draw the ray OG, which is the bisector of $\angle B'OF$. Hence, $\angle GOA$ is the required angle of $150°$.

Justification :

$$\angle B'OG = \angle FOG = \frac{1}{2}\angle B'OF$$

$$= \frac{1}{2}(60°) = 30°$$

Thus, $$\angle GOA = \angle FOG + \angle FOE + \angle EOA$$
$$= 30° + 60° + 60° = 150°$$

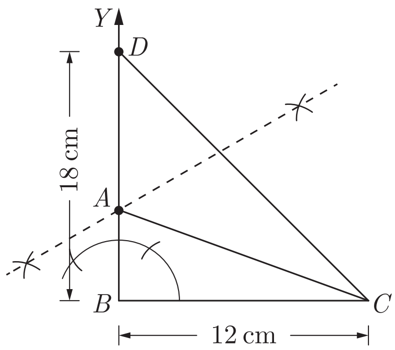
On measuring the $\angle GOA$ by protractor, we find that $\angle GOA = 150°$. Thus, the construction is justified.

or

Construct a right triangle whose base is 12 cm and sum of its hypotenuse and other side is 18 cm.

SOLUTION :

Steps of Construction :
(i) Draw $BC = 12\,\text{cm}$.
(ii) Construct $\angle CBY = 90°$.
(iii) From ray BY, cut-off line segment $BD = 18\,\text{cm}$.



(iv) Join CD.
(v) Draw the perpendicular bisector of CD intersecting BD at A.
(vi) Join AC to obtain the required ΔABC.
Justification :
Since A lies on the perpendicular bisector of CD. Therefore,

$$AD = AC$$

Now, $$BD = BA + AD$$

$\Rightarrow$ $$BD = AB + AC$$

Hence, ΔABC is the required triangle.

39. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6m ? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (Take $\pi = 3.14$) [4]

SOLUTION :

Let r, h and l be the radius, height and slant height of the tent, respectively.
Given, $r = 6$ m and $h = 8$ m
We know that,

$$l^2 = h^2 + r^2$$

$\Rightarrow$ $$l = \sqrt{r^2 + h^2}$$

[On taking positive square root]

$$l = \sqrt{(6)^2 + (8)^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100} = 10\text{ m}$$

Area of the canvas used for the tent

= Curved surface area of the cone
$$= \pi rl = 3.14 \times 6 \times 10$$
$$= 188.4\text{ m}^2$$

$\therefore$ Length of tarpaulin required

$$= \frac{\text{Area of tarpaulin required}}{\text{Width of tarpaulin}}$$

$$= \frac{188.4}{3} = 62.8\text{ m}$$

The extra material required for stitching margins and cutting $= 20\text{ cm} = 0.2\text{ m}$ $\left[\because 1\text{ cm} = \frac{1}{100}\text{ m}\right]$

Hence, the total length of tarpaulin required

$$= 62.8 + 0.2 = 63\text{ m}$$

40. Cards marked with the numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is : [4]
(i) an even number
(ii) a number less than 14
(iii) a number which is a perfect square.

SOLUTION :

Since, the cards are marked from 2 to 101.
Therefore, total number of cards = 100
(i) There are 50 cards marked with even numbers from 2 to 101.

$\therefore$ $$P\text{ (getting an even number)} = \frac{50}{100} = \frac{1}{2}$$

Hence, the probability of getting even number card is $\frac{1}{2}$.
(ii) There are 12 cards on which marked numbers are less than 14.

$\therefore$ $$P\text{ (getting a number less than 14)} = \frac{12}{100} = \frac{3}{25}$$

Hence, the probability that the number on card is less than 14, is $\frac{3}{25}$.
(iii) The number from 2 to 101 which are perfect square are 4, 9, 16, 25, 36, 49, 64, 81, 100 i.e. Square of 2, 3, 4, 5, 6, 7, 8, 9 and 10 respectively.
Total number of cards having a number which is a perfect square = 9
$\therefore$ P (getting a number which is a perfect square)
$$= \frac{9}{100}$$
Hence, the probability that the number marked on the card which is a perfect square, is $\frac{9}{100}$.

or

The average weight of all male stars in a multi-star Bollywood movie is 71.2 kg where as average weight of all female co-stars is 50.8 kg. If the mean weight of male and female stars acting in the movie is 60 kg. Find the ratio of number of male stars to the number of female co-stars acting in the movie.

SOLUTION :

Let n_1 and n_2 be number of male and female stars acting in Bollywood movie.

We have $\overline{x}_1 = 71.2\,\text{kg}$

$\overline{x}_2 = 58.8\,\text{kg}$

and $\overline{x} = 60\,\text{kg}$

We know that, $\overline{x} = \dfrac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$

$$\Rightarrow \quad 60 = \frac{n_1(71.2) + n_2(50.8)}{n_1 + n_2}$$

$$60(n_1 + n_2) = 71.2\,n_1 + 50.8\,n_2$$

$$60\,n_2 - 50.8\,n_2 = 71.2\,n_1 - 60\,n_1$$

$$9.2\,n_2 = 11.2\,n_1$$

$$\frac{n_1}{n_2} = \frac{9.2}{11.2} = \frac{23}{28}$$

Hence, the ratio of number of male stars to female stars in the movie is 23 : 28.