CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-5

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The value of $\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$ is equal to [1]

(a) $x^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$ (b) 0

(c) $x^{pq+qr+rp}$ (d) 1

Ans : (d) 1

$$\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}} = \frac{x^{\frac{1}{r}}}{x^{\frac{1}{q}}} \times \frac{x^{\frac{1}{p}}}{x^{\frac{1}{r}}} \times \frac{x^{\frac{1}{q}}}{x^{\frac{1}{p}}} = 1$$

2. For the polynomial $p(x) = x^5 + 4x^3 - 5x^2 + x - 1$, one of the factors is [1]

(a) $(x+1)$ (b) $(x-1)$

(c) x (d) $(x+2)$

Ans : (b) $(x-1)$

$$p(x) = x^5 + 4x^3 - 5x^2 + x - 1$$
$$p(1) = 1 + 4 - 5 + 1 - 1 = 0$$

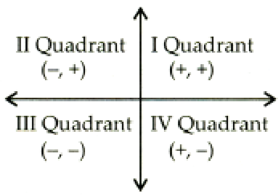
Hence, $x = 1$ is the solution of $p(x)$.

3. The point for which the abscissa and ordinate have same signs will lie in [1]

(a) I and II quadrants (b) I and III quadrants

(c) I and IV quadrants (d) III and IV quadrants

Ans : (b) I and III quadrants



Abscissa and ordinate have same sign in I and III quadrants.

4. Which of the following equation has graph parallel to y-axis? [1]

(a) $y = -2$ (b) $x = 1$

(c) $x - y = 2$ (d) $x + y = 2$

Ans : (b) $x = 1$

$x = a$ has the graph which is parallel to y-axis.
$x = 1$ is the required equation that has graph parallel to y-axis.

5. Axioms are [1]

(a) universal truths in all branches of Mathematics

(b) universal truths specific to geometry

(c) theorems

(d) definitions

Ans : (a) universal truths in all branches of Mathematics

Axioms are universal truths in all branches of Mathematics.

6. If two parallel lines are intersected by a transversal, then each pair of corresponding angles so formed is [1]

(a) Equal (b) Complementary

(c) Supplementary (d) None of these

Ans : (a) Equal

7. Which of the following is a correct statement? [1]

(a) In an isosceles triangle, the angles opposite to equal sides are equal.

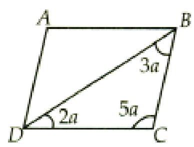
(b) If the hypotenuse and an acute angle of the right-angled triangle are not equal to the hypotenuse and the corresponding acute angle of another triangle, then the triangles are congruent.

(c) The bisector of the vertical angle of an isosceles triangle bisects the base at acute angles.

(d) All of these

Ans : (a) In an isosceles triangle, the angles opposite to equal sides are equal.

8. In the given figure, the measure of $\angle C$ is equal to [1]



(a) 90° (b) 80°

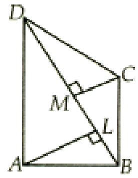
(c) 75° (d) 95°

Ans : (a) 90°

In, ΔBCD $\angle BCD + \angle CDB + \angle DBC = 180^\circ$

$$2a + 5a + 3a = 180^\circ$$
$$a = 18^\circ$$
$$\angle C = 5a$$
$$= 5 \times 18^\circ = 90^\circ$$

9. In the adjoining figure, $ABCD$ is a quadrilateral in which diagonal $BD = 14\,\text{cm}$. If $AL \perp BD$ and $CM \perp BD$ such that $AL = 8\,\text{cm}$ and $CM = 6\,\text{cm}$, then area of quadrilateral $ABCD$ is [1]



(a) 60 cm^2 (b) 72 cm^2
(c) 84 cm^2 (d) 98 cm^2

Ans : (d) 98 cm^2

Area of quadrilateral $ABCD$

$$= \text{area}(\Delta ABD) + (\Delta BCD)$$
$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$
$$= \left[\frac{1}{2} \times 14 \times 8 + \frac{1}{2} \times 14 \times 6\right]\text{cm}^2$$
$$= 98\,\text{cm}^2$$

10. Which of the following statements is true for a regular pentagon? [1]
(a) All vertices are con-cyclic.
(b) All vertices are not con-cyclic.
(c) Only four vertices are con-cyclic
(d) Cannot say anything about regular pentagon

Ans : (a) All vertices are con-cyclic.

(Q.11-Q.15) Fill in the blanks :

11. The construction of a triangle ABC, given that $BC = 3\,\text{cm}$, $\angle C = 60^\circ$ is possible when difference of AB and AC is equal to .... cm [1]

Ans : 2.8 cm

A triangle can be constructed when difference of two of its sides is less than the third side.

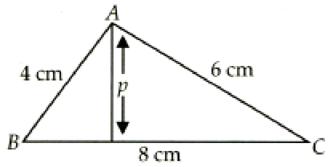
12. The length of the sides of a triangle are 4 cm, 6 cm and 8 cm. The length of perpendicular from the opposite vertex to the side whose length is 8 cm, is equal to .... cm. [1]

Ans : $\frac{3}{4}\sqrt{15}\,\text{cm}$

$$s = \frac{1}{2}(4 + 6 + 8)\,\text{cm} = 9 \text{ cm}$$
$$\text{Area} = \sqrt{9(9-4)(9-6)(9-8)}$$
$$= \sqrt{9 \times 5 \times 3 \times 1}$$
$$= 3\sqrt{15}\text{ cm}^2$$

Also, $\text{area} = \frac{1}{2} \times 8 \times p$

$$4p = 3\sqrt{15}$$
$$p = \frac{3\sqrt{15}}{4}\,\text{cm}$$



or

Area of a triangle with perimeter 42 cm and length of two sides 18 cm and 10 cm is given by ....

Ans : $21\sqrt{11}\text{ cm}^2$

13. A sphere has only .... surface and that is curved.[1]

Ans : One

14. If n is an odd number, the median = value of the .... observation. [1]

Ans : $\left(\frac{n+1}{2}\right)^{\text{th}}$

15. Number of favourable outcomes for an event cannot be .... than the number of total outcomes. [1]

Ans : Greater

(Q.16-Q.20) Answer the following :

16. The hollow sphere, in which the circus motorcyclist performs his stunt, has a diameter of $7\,\text{m}$. Find the area available to the motorcyclist for riding? [1]

SOLUTION :

Given,

Diameter of the sphere $= 7\,\text{m}$.

Therefore, radius is $3.5\,\text{m}$, So, the riding space available for the motorcyclist is the surface area ot the sphere.

$$4\pi r^2 = 4 \times \frac{22}{7} \times 3.5 \times 3.5\,\text{m}^2$$
$$= 154\,\text{m}^2$$

17. Find k, if $x^{51} + 2x^{60} + 3x + k$ is divisible by $x + 1$. [1]

SOLUTION :

Let, $p(x), = x^{51} + 2x^{60} + 3x + k$

Given that, $p(x)$, is divisible by $x + 1$.

$$p(-1) = 0$$
$$(-1)^{51} + 2(-1)^{60} + 3(-1) + k = 0$$
$$-1 + 2 - 3 + k = 0$$
$$k - 4 + 2 = 0$$
$$k - 2 = 0$$
$$k = 2$$

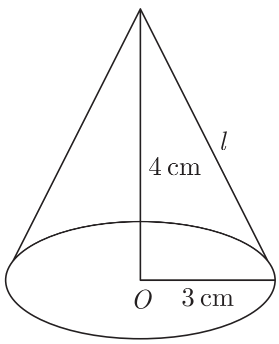
18. Which of the following points lies in II-quadrant. [1]
$A(2,3), B(-2,6), C(-2,-3), D(-1,2), E(4,1)$.

SOLUTION :

Points lies in II-quadrant are point B and point D.

19. The radius of a cone is $3\,\text{cm}$ and vertical heights is $4\,\text{cm}$. Find the area of the curved surface. [1]

SOLUTION :



We have, $r = 3\text{ cm}$ and $h = 4\text{ cm}$.
Let l cm be the slant height of the cone.

Then, $l^2 - r^2 + h^2$

$l^2 = \sqrt{25}\text{ cm} = 5\text{ cm}$

Area of the curved surface $= \pi rl$

Area of the curved surface $= \left(\frac{22}{7} \times 4 \times 5\right)\text{cm}^2$

$= 62.85\text{ cm}^2$

20. Find the probability of Sun revolving around Earth.[1]

SOLUTION :

Here, it is an impossible event, because we know that Earth revolves around the Sun. Also, it is universal truth.
Probability of the Sun revolving around Earth = 0.

Section B

21. Simplify : $\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$. [2]

SOLUTION :

We have

$$\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \frac{\sqrt[3]{6^2} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$$

$$= \frac{\sqrt[3]{6^2 \times 6^7}}{\sqrt[3]{6^6}} \quad [\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}]$$

$$= \frac{\sqrt[3]{6^9}}{\sqrt[3]{6^6}} \quad [\because a^m \times a^n = (a)^{m+n}]$$

$$= \sqrt[3]{\frac{6^9}{6^6}} = \sqrt[3]{6^{9-6}}$$

$$\left[\because \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \text{ and } a^m + a^n = (a)^{m-n}\right]$$

$$= \sqrt[3]{6^3} = 6 \quad [\because \sqrt[m]{a^m} = a]$$

or

If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$, find the values of a and b.

SOLUTION :

We have,

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$$

$$\Rightarrow \quad \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = a + b\sqrt{3}$$

$$\frac{35 - 20\sqrt{3} + 14\sqrt{3} - 24}{49 - 48} = a + b\sqrt{3}$$

$$11 - 6\sqrt{3} = a + b\sqrt{3}$$

$$\Rightarrow \quad a = 11 \text{ and } b = -6$$

22. If $\left(x + \frac{1}{x}\right) = 9$, then find the value of $x^3 + \frac{1}{x^3}$. [2]

SOLUTION :

We have, $x + \frac{1}{x} = 9$

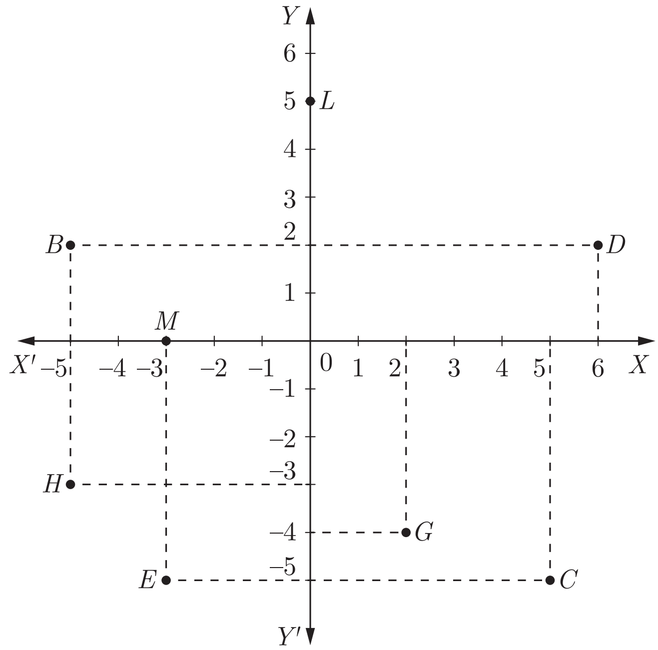
$$\left(x + \frac{1}{x}\right)^3 = 9^3$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 729$$

$$x^3 + \frac{1}{x^3} + 3 \times 9 = 729$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 729 - 27 = 702$$

23. See Fig. and write the following : [2]



(i) The coordinates of B.
(ii) The coordinates of C.
(iii) The point identified by the coordinates (–3, –5).
(iv) The point identified by the coordinates (2, –4).

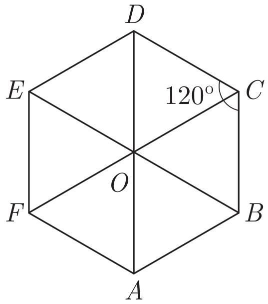
SOLUTION :

(i) (–5, 2)
(ii) (5, –5)
(iii) E
(iv) G

24. Find the area of regular hexagon of side a cm. [2]

SOLUTION :

We know that, regular hexagon is divided into six equilateral triangles.



$\therefore$ Area of regular hexagon of side a

$= $ Sum of the area of six equilateral triangles

$$= 6 \times \frac{\sqrt{3}}{4} \times a^2 = \frac{3\sqrt{3}}{2} a^2 \text{ cm}^2$$

[$\because$ Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$]

or

The sides of a triangle are 4 cm, 8 cm and 6 cm. Find the length of the perpendicular from the opposite vertex to the longest side.

SOLUTION :

$$s = \frac{4+8+6}{2} \text{ cm} = 9 \text{ cm}$$

$\therefore$Area of the triangle$= \sqrt{9(9-4)(9-8)(9-6)}$ cm^2

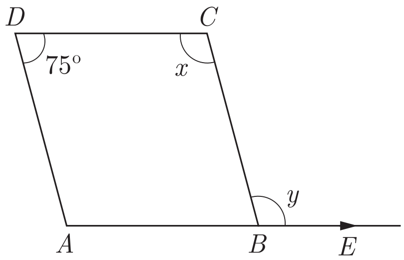
$$= \sqrt{9 \times 5 \times 1 \times 3} \text{ cm}^2$$

$$= 3\sqrt{15} \text{ cm}^2$$

Also, $\frac{1}{2} \times 8 \times \text{Altitude} = 3\sqrt{15}$

$$\text{Altitude} = \frac{3\sqrt{15}}{4} \text{ cm}$$

25. $ABCD$ is a parallelogram in which $\angle ADC = 75^\circ$ and side AB is produced to point E as shown in the figure. Find $(x+y)$. [2]



SOLUTION :

Given, $ABCD$ is a parallelogram, in which $\angle ADC = 75^\circ$

$\therefore$ $\angle ABC = 75^\circ$

[In a parallelogram, opposite sides are equal]

$\angle CBE = y = 180^\circ - \angle ABC$

[Linear pair axiom]

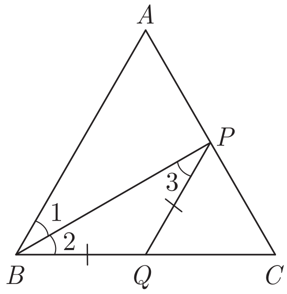
$$= 180^\circ - 75^\circ = 105^\circ$$

Also, $\angle x = 180^\circ - 75^\circ = 105^\circ$

[$\because$ $\angle D + \angle x = 180^\circ$ as $DA || CB$ and DC is a transversal]

$\therefore$ $x + y = 105^\circ + 105^\circ = 210^\circ$

26. P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle. [2]



SOLUTION :

Given : BP is bisector of $\angle ABC$ and $PQ || AB$.
To Prove : ΔBPQ is an isosceles triangle.
Proof :

$\angle 1 = \angle 2$ ...(1) [BP bisects $\angle B$]

$\angle 3 = \angle 1$ ...(2) [Alternate angles]

From (1) and (2), we have

$\Rightarrow$ $\angle 2 = \angle 3$

Then, $PQ = BQ$

[Sides opposite to equal angles of a ΔBPQ]
Hence, ΔBPQ is an isosceles triangle.

or

In quadrilateral $PQRS$, if $\angle P = 60^\circ$ and $\angle Q : \angle R : \angle S = 2:3:7$, then find the value of $\angle S$.

SOLUTION :

Let the angle Q be $2x$, angle R be $3x$ and angle S be $7x$.

Then, $\angle P + \angle Q + \angle R + \angle S = 360^\circ$

$\Rightarrow$ $60^\circ + 2x + 3x + 7x = 360^\circ$

$$60^\circ + 12x = 360^\circ$$

$$12x = 360^\circ - 60^\circ$$

$$= 300^\circ$$

$$x = \frac{300^\circ}{12} = 25^\circ$$

$\therefore$ $\angle S = 7x = 7 \times 25^\circ = 175^\circ$

Section C

27. Find the remainder, when $3x^3 - 6x^2 + 3x - \frac{7}{9}$ is divided by $3x - 4$. [3]

SOLUTION :

Let $p(x) = 3x^3 - 6x^2 + 3x - \frac{7}{9}$ and it is divided by $3x - 4$.

Put $3x - 4 = 0$

$\Rightarrow$ $3x = 4$

$$x = \frac{4}{3}$$

On putting $x = \frac{4}{3}$ in $p(x)$, we get

$$p\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^3 - 6\left(\frac{4}{3}\right)^2 + 3\left(\frac{4}{3}\right) - \frac{7}{9}$$

$$= 3 \times \left(\frac{64}{27}\right) - 6 \times \left(\frac{16}{9}\right) + 4 - \frac{7}{9}$$

$$= \frac{64}{9} - \frac{32}{3} + 4 - \frac{7}{9}$$

$$= \frac{64 - 96 + 36 - 7}{9}$$

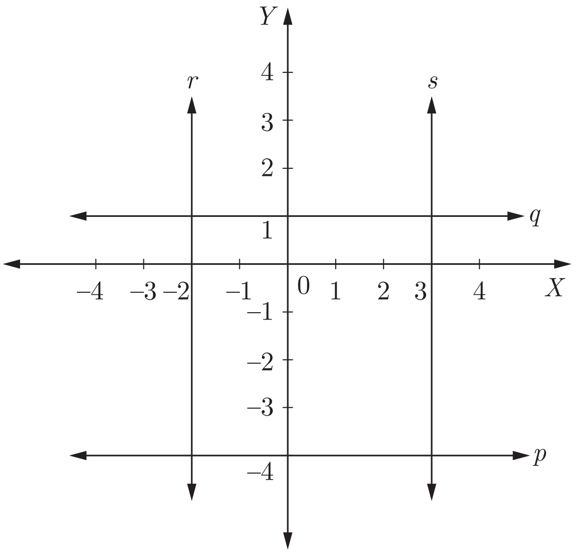
$$= -\frac{3}{9} = -\frac{1}{3}$$

Hence, the remainder is $-\frac{1}{3}$.

or

Write the equation of the lines drawn in following graph. Also, find the area enclosed between them.

SOLUTION :



The line q is parallel to X-axis and at 1 unit distance from X-axis in the positive direction of Y-axis.

So, equation of line q is $y = 1$.

The line p is parallel to X-axis and at 4 units distance from X-axis in the negative direction of Y-axis.

So, the equation of line p is $y = -4$.

Again, the line r is parallel to X-axis and at a distance of 2 units from Y-axis in the negative direction of X -axis.

So, the equation of line r is $x = -2$.

Similarly, the equation of line s is $x = 3$.

Thus, we get the equation of lines as $y = 1$, $y = -4$, $x = -2$, $x = 3$.

Thus, formed figure by these lines is of a square of length 5 units.

$\therefore$ Area of formed figure $= 5 \times 5 = 25$ sq units.

28. A family with monthly income of ₹ 30,000 had planned the following expenditures per month under various heads : [3]

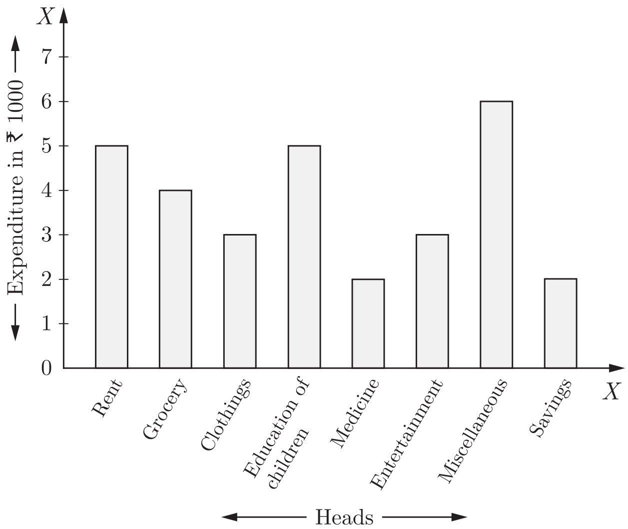
Heads	**Expenditure (in ₹ 1000)**
Rent	5
Grocery	4
Clothings	3
Education of children	5
Medicine	2
Entertainment	3
Miscellaneous	6
Savings	2

Draw a bar graph for the above data.

SOLUTION :

Let us take heads along x-axis and expenditure (in ₹ 1000) among y-axis.

Along y-axis take 1 big division = ₹ 1000



or

If the mean of five observations x, $x + 2$, $x + 4$, $x + 6$ and $x + 8$ is 11. Find the value of x.

SOLUTION :

Mean of the given observations

$$= \frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8)}{5}$$

$$= \frac{5x + 20}{5}$$

But mean = 11 [Given]

$$\therefore \quad \frac{5x + 20}{5} = 11$$

$$\Rightarrow \quad 5x + 20 = 55$$

$$x = 7$$

Hence, $x = 7$

29. Find the curved surface area and total surface area of a hemisphere of radius 35 cm. [3]

SOLUTION :

Here, radius of the hemisphere $(r) = 35$ cm

$\therefore$ Curved surface area of hemisphere $= 2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times 35 \times 35\right)\text{cm}^2$$

$$= 44 \times 5 \times 35 = 7700 \text{ cm}^2$$

Total surface area of hemisphere $= 3\pi r^2$

$$= \left(3 \times \frac{22}{7} \times 35 \times 35\right) \text{cm}^2$$

$$= 66 \times 5 \times 35 = 11550 \text{ cm}^2$$

or

The volume of a cylindrical rod is 628 cm^3. If its height is 20 cm, find the radius of its cross section. (Use $\pi = 3.14$).

SOLUTION :

Let radius of cross section of rod $= r$ cm
Height of cylindrical rod $= 20$ cm
Volume of cylindrical rod $= 628 \text{ cm}^3$

$$\Rightarrow \quad \pi r^2 h = 628$$

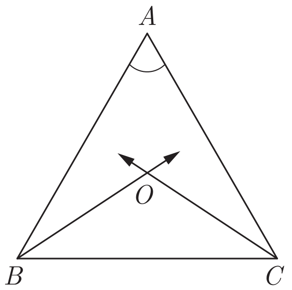
$$3.14 \times r^2 \times 20 = 628$$

$$r^2 = \frac{628 \times 100}{314 \times 20} = 10$$

$$r = \sqrt{10} \text{ cm} = 3.16 \text{ cm}$$

$\therefore$ Radius of its cross section $= 3.16$ cm

30. In the given figure, the bisectors of $\angle ABC$ and $\angle BCA$, intersect each other at point O. If $\angle BOC = 100°$, then find $\angle A$. [3]



SOLUTION :

In ΔABC, we have

$$\angle A + \angle B + \angle C = 180°$$

[Since, sum of all the angles of a triangle is 180°]

$$\Rightarrow \quad \angle B + \angle C = 180° - \angle A$$

$$\frac{\angle B}{2} + \frac{\angle C}{2} = 90° - \frac{\angle A}{2}$$

[Dividing both sides by 2]

$$\angle OBC + \angle OCB = 90° - \frac{\angle A}{2} \quad ...(1)$$

[Since, OB and OC are the bisectors of angles B and C]

Now, in ΔOBC, we have

$$\angle OBC + \angle OCB + \angle BOC = 180°$$

[Since, sum of all the angles of a triangle is 180°]

$$\Rightarrow \quad \angle BOC = 180° - [\angle OBC + \angle OCB]$$

$$100° = 180° - \left[90° - \frac{\angle A}{2}\right]$$

[$\because$ $\angle BOC = 100°$, given and from eq.(1)]

$$100° = 180° - 90° + \frac{\angle A}{2}$$

$$100° = 90° + \frac{\angle A}{2}$$

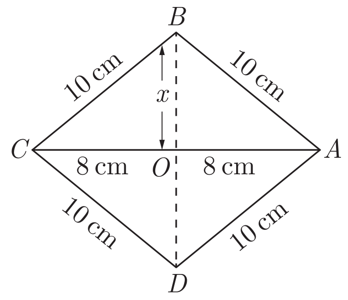
$$\frac{\angle A}{2} = 100° - 90° = 10°$$

$$\therefore \quad \angle A = 20°$$

31. Write true or false and justify your answer. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is 96 cm^2. [3]

SOLUTION :

True. We know that diagonals of a rhombus bisect each other at right angle.



$$\therefore \quad OA = OC = 8 \text{ cm}$$

In ΔOAB,

$$AB^2 = OA^2 + OB^2$$

$$(10)^2 = (8)^2 + (x)^2$$

$$x = \sqrt{36} = 6 \text{ cm}$$

$$DB = 2(OB)$$

$$= 2 \times 6 = 12 \text{ cm}$$

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}^2$$

32. $ABCD$ is a parallelogram. A circle through A and B is drawn, so that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic. [3]

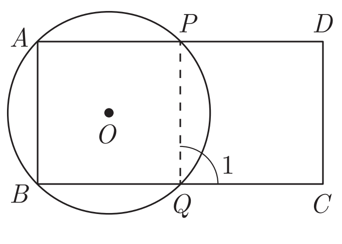
SOLUTION :

Here, join PQ.

Now, $\quad \angle 1 = 180° - \angle BQP$

$\Rightarrow \quad \angle 1 = \angle A$

[By property of cyclic quadrilateral]



But $\quad \angle A = \angle C$

[Opposite angles of a parallelogram]

$\therefore \quad \angle 1 = \angle C \quad ...(1)$

But $\quad \angle C + \angle D = 180°$

[Sum of co-interior angles on same side is 180°]
[From eq.(1)]

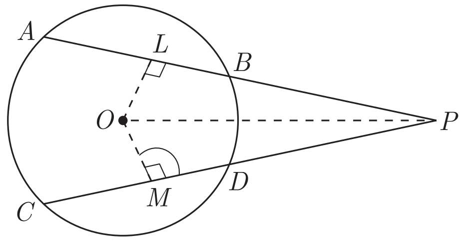
$\Rightarrow \quad \angle 1 + \angle D = 180°$

Thus, the quadrilateral $QCDP$ is cyclic.
So, the points P, Q, C and D are concyclic.

33. Two equal chords AB and CD of a circle when produced, intersect at a point P. Prove that $PB = PD$. [3]

SOLUTION :

Given : Two equal chords AB and CD of a circle intersecting at a point P.



To prove : $PB = PD$

Construction : Join OP. Draw $OL \perp AB$ and $OM \perp CD$

Proof : Since, equal chords are equidistant from the centre.

$\therefore \qquad OL = OM$

In ΔOLP and ΔOMP,

$OL = OM$ [Proved above]

$\angle OLP = \angle OMP$ [Each 90°]

$OP = OP$ [Common sides]

$\therefore \qquad \Delta OLP \cong \Delta OMP$ [By RHS congruence rule]

$\Rightarrow \qquad LP = MP$ ...(1) [By CPCT]

Now, $\qquad AB = CD$

$\Rightarrow \qquad \frac{1}{2}(AB) = \frac{1}{2}(CD)$ [Dividing by 2]

$BL = DM$ ...(2)

[$\because$ Perpendicular drawn from centre to the circle bisects the chord i.e., $AL = LB$ and $CM = MD$]

On subtracting eq. (2) from eq. (1), we get

$\Rightarrow \qquad LP - BL = MP - DM$

$PB = PD$

Hence proved.

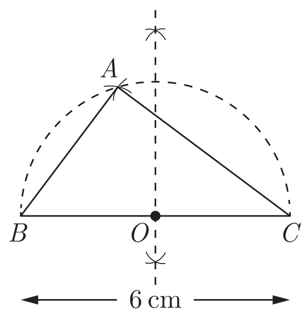
34. Draw a right angled triangle whose hypotenuse measure 6 cm and the length of one of whose sides containing the right angle is 4 cm. [3]

SOLUTION :

Steps of construction :

(i) Draw a line segment $BC = 6\,\text{cm}$.

(ii) Draw perpendicular bisector of BC which intersects BC at O.



(iii) Taking O as centre and radius OB, draw a semi-circle on BC.

(iv) Taking B as centre and radius equal to 4 cm. draw an arc, cutting the semi-circle at A.

(v) Join AB and AC.

Thus, ABC is the required right and angled triangle.

Section D

35. A recent survey found that the age of workers in a factory as follows : [4]

Age (in yrs)	Number of workers
20-29	38
30-39	27
40-49	86
50-59	46
60 and above	3

If a person is selected at random, then find the probability that the person is

SOLUTION :

Total number of workers in a factory,

$$n(S) = 38 + 27 + 86 + 46 + 3 = 200$$

(i) Number of persons having age of 40 yrs or more,

$$n(E_1) = 86 + 46 + 3 = 135$$

$\therefore$ Probability that the person selected at the age of 40 yrs or more.

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{135}{200} = 0.675$$

Hence, the probability that the person selected at the age of 40 yrs or more is 0.675.

(ii) Number of persons under the age of 40 yrs.

$$n(E_2) = 38 + 27 = 65$$

$\therefore$ Probability that the selected person under the age of 40 yrs,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{65}{200} = 0.325$$

Hence, the probability that the selected person under 40 yrs is 0.325.

(iii) Number of persons having age from 30 to 39 yrs,

$$n(E_3) = 27$$

$\therefore$ Probability that the selected person have age from 30 to 39 yrs.

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{27}{200} = 0.135$$

Hence, the probability that the selected person have age from 30 to 39 yrs is 0.135.

or

The mean of the following frequency distribution is 16.6.

x_i	8	12	15	18	20	25	30	Total
f_i	12	16	p	24	16	q	4	100

Find the missing frequencies p and q.

SOLUTION :

We prepare the table given below :

x_i	f_i	$f_i x_i$
8	12	96
12	16	192
15	p	$15p$
18	24	432
20	16	320
25	q	$25q$
30	4	120
	$\Sigma f_i = 72+p+q$	$\Sigma f_i x_i = 1160+15p+25q$

Here, $\Sigma f_i = 72+p+q$

But, $\Sigma f_i = 100$ (Given)

$\therefore$ $72+p+q = 100$

$p+q = 28$

Also, Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$= \dfrac{1160+15p+25q}{72+p+q}$

$= \dfrac{1160+15(p+q)+10q}{72+(p+q)}$

$= \dfrac{1160+15\times 28+10q}{72+28}$

$= \dfrac{1580+10q}{100}$

But mean $= 16.6$ (Given)

$\therefore$ $\dfrac{1580+10q}{100} = 16.6$

$1580+10q = 1660$

$10q = 80$

$q = 8$

$\Rightarrow$ $p+q = 28$

$p = 28-q$

$= 28-8 = 20$

Hence, $p = 20$ and $q = 8$.

36. If $x = \dfrac{1}{2-\sqrt{3}}$, then find the value of x^3-2x^2-7x+5 . [4]

SOLUTION :

Given, $x = \dfrac{1}{2-\sqrt{3}} = \dfrac{1}{2-\sqrt{3}}\times\dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

[By rationalising the denominator]

$= \dfrac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \dfrac{2+\sqrt{3}}{4-3}$

$= 2+\sqrt{3}$

$[\because (a-b)(a+b) = a^2-b^2]$

$\Rightarrow$ $x-2 = \sqrt{3}$

On squaring both sides, we get

$(x-2)^2 = (\sqrt{3})^2$

$\Rightarrow$ $x^2-4x+4 = 3$ $[\because (a-b)^2 = a^2-b^2-2ab]$

$x^2-4x+1 = 0$ ...(1)

Now, divide (x^3-2x^2-7x+5) by (x^2-4x+1).

$$\begin{array}{r|l} & x+2 \\ x^2-4x+1 & x^3-2x^2-7x+5 \\ & x^3-4x^2+x \\ & -\quad+\quad- \\ \hline & 2x^2-8x+5 \\ & 2x^2-8x+2 \\ & -\quad+\quad- \\ \hline & 3 \end{array}$$

By using long division method, we get

Thus, quotient $= x+2$ and remainder $= 3$

$\therefore$ $x^3-2x^2-7x+5 = (x+2)(x^2-4x+1)+3$

$= 0+3 = 3$ [Using eq.(1)]

Hence, at $x = \dfrac{1}{2-\sqrt{3}}$, $x^3-2x^2-7x+5 = 3$

37. Water flows in a tank 150 m $\times$ 100 m at the base through a pipe whose cross-section is 2 dm $\times$ 1.5 dm at the speed of 15 km/h. In what time, will the water be 3 m deep ? [4]

SOLUTION :

Suppose in x hours water will be 3 m deep in tank.

Volume of water in the tank

$= 150\times 100\times 3 = 45000\,\text{m}^3$

Area of cross-section of the pipe

$= \dfrac{2}{10}\times\dfrac{1.5}{10} = \dfrac{1}{5}\times\dfrac{15}{100}$

$= \dfrac{3}{100}\,\text{m}^2$ $\left[\therefore 1\,\text{dm} = \dfrac{1}{10}\text{m}\right]$

Volume of water that flows in the tank in x hours

$=$ Area of cross-section of the pipe

$\times$ Speed of water $\times$ Time

$= \dfrac{3}{100}\times 15000\times x$

$[\because \text{speed} = 15\,\text{km/h} = 15000\,\text{m/h}]$

$= 450x\,\text{m}^3$

Since, the volume of water in the tank is equal to the volume that flows in the tank in x hours.

$\therefore$ Volume of water in the tank

$=$ Volume of water that flows in x hours

$\therefore$ $450x = 45000\,\text{h}$

$x = 100$ hours

or

An open rectangular cistern is made of iron 2.5 cm thick. When measured from outside, it is 1 m 25 cm long, 1 m 5 cm broad and 90 cm deep.

Find :

(i) the capacity of the cistern in litres

(ii) the volume of iron used

(iii) the total surface area of the cistern

SOLUTION :

External dimensions of the cistern are :

$$\text{Length} = 125\text{ cm}$$
$$\text{Breadth} = 105\text{ cm}$$
and $$\text{Depth} = 90\text{ cm}$$

Internal dimensions of the cistern are :

$$\text{Length} = 120\text{ cm}$$
$$\text{Breadth} = 100\text{ cm}$$
and $$\text{Depth} = 87.5\text{ cm}$$

(i) $$\text{Capacity} = \text{Internal volume}$$
$$= (120 \times 100 \times 87.5)\text{cm}^3$$
$$= \left(\frac{120 \times 100 \times 87.5}{1000}\right)$$
$$= 1050 \text{ litres}$$

(ii) $$\text{Volume of iron} = (\text{External volume}) - (\text{Internal volume})$$
$$= \{(125 \times 105 \times 90) - (120 \times 100 \times 87.5)\}$$
$$= (1181250 - 1050000)$$
$$= 131250\text{ cm}^3$$

(iii) $$\text{External area} = (\text{Area of 4 faces}) + (\text{Area of the base})$$
$$= \{[2(125 + 105) \times 90] + (125 \times 105)\}$$
$$= (41400 + 13125)$$
$$= 54525\text{ cm}^2$$
$$\text{Internal area} = \{[2(120 + 100) \times 87.5] + (120 \times 100)\}$$
$$= (38500 + 12000)$$
$$= 50500\text{ cm}^2$$

Area at the top = Area between outer and inner rectangles
$$= \{(125 \times 105) - (120 \times 100)\}$$
$$= (13125 - 12000)$$
$$= 1125\text{ cm}^2$$
$$\therefore \text{Total surface area} = (54525 + 50500 + 1125)$$
$$= 106150\text{ cm}^2$$

38. Find the zeroes of the given polynomial $f(x) = 2x^3 + 3x^2 - 11x - 6$. [4]

SOLUTION :

We have $$f(x) = 2x^3 + 3x^2 - 11x - 6$$
Here, the constant term is 6. Then, the factors of 6 may be ± 1, ± 2, ± 3 and ± 6.
By trial method, put $x = -1$ in $f(x)$, we get
$$f(-1) = 2(-1)^3 + 3(-1)^2 - 11(-1) - 6$$
$$= -2 + 3 + 11 - 6 \neq 0$$
Thus, $x = -1$ is not a zero of $f(x)$.
Now, put $x = 2$ in $f(x)$, we get
$$f(2) = 2(2)^3 + 3(2)^2 - 11(2) - 6$$
$$= 16 + 12 - 22 - 6$$
$$= 28 - 28 = 0$$

$\therefore$ $x = 2$ is zero of $f(x)$.
$\Rightarrow$ $(x-2)$ is a factor of $f(x)$.
Then, $f(x) = (x-2) \cdot q(x)$, where $q(x)$ is a quadratic polynomial of degree 2, which is obtained on dividing, $f(x)$ by $(x-2)$ by using long division method.

$$\begin{array}{r|l} & 2x^2 + 7x + 3 \\ \hline x-2 & 2x^3 + 3x^2 - 11x - 6 \\ & 2x^3 - 4x^2 \\ \hline & 7x^2 - 11x \\ & 7x^2 - 14x \\ \hline & 3x - 6 \\ & 3x - 6 \\ \hline & 0 \end{array}$$

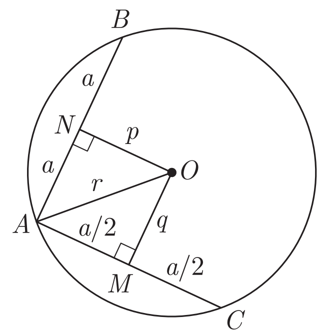
Thus, $$q(x) = 2x^2 + 7x + 3$$
$$\therefore \quad f(x) = (x-2)(2x^2 + 7x + 3)$$
$$= (x-2)(2x^2 + 6x + x + 3)$$
[By spliting the middle term]
$$= (x-2)[2x(x+3) + 1(x+3)]$$
$$= (x-2)(x+3)(2x+1)$$
Thus, $f(x) = 0$
if $(x-2) = 0$
or $(x+3) = 0$ or $(2x+1) = 0$
$\Rightarrow$ $x = 2$
$x = -3$
$x = -\frac{1}{2}$

Hence, 2, –3 and $-\frac{1}{2}$ are zeroes of the given polynomial.

39. AB and AC are two chords of a circle of radius r such that $AB = 2AC$. If p and q are the distances of AB and AC from the centre then prove that $4q^2 = p^2 + 3r^2$. [4]

SOLUTION :

Let $AC = a$, then $AB = 2a$
From centre O, perpendicular is drawn to the chords AC and AB at M and N, respectively.
$$\therefore \quad AM = MC = \frac{a}{2}$$
and $$AN = NB = a$$
In ΔOMA and ΔONA,



By Pythagoras theorem,
$$AO^2 = AM^2 + MO^2$$
$$\Rightarrow \quad AO^2 = \left(\frac{a}{2}\right)^2 + q^2 \quad ...(1)$$

and $AO^2 = (AN)^2 + (NO)^2$

$\Rightarrow$ $AO^2 = (a)^2 + p^2$ ...(2)

From eqs.(1) and (2), we get

$$\left(\frac{a}{2}\right)^2 + q^2 = a^2 + p^2$$

$$\Rightarrow \quad \frac{a^2}{4} + q^2 = a^2 + p^2$$

$$a^2 + 4q^2 = 4a^2 + 4p^2$$

$$4q^2 = 3a^2 + 4p^2$$

$$4q^2 = p^2 + 3(a^2 + p^2)$$

$$4q^2 = p^2 + 3r^2$$

[$\because$ in right angled ΔONA, $r^2 = a^2 + p^2$]

40. A man hires an auto rickshaw to cover a certain distance. The fare is ₹10 for first kilometre and ₹7 for subsequent kilometres. Taking total distance covered as x km and total fare as ₹ y. [4]

(i) Write a linear equation for this.

(ii) The man covers a distance of 16 km and gave ₹120 to the auto driver. Auto driver said. "it is not the correct amount" and returned him the balance. Find the correct amount paid back by the auto driver.

SOLUTION :

(i) Given,

total distance covered $= x$ km

$= 1 + (x-1)$ km

and total fare $=$ ₹ y

Fare for first kilometre $=$ ₹10

Fare for subsequent kilometres $=$ ₹7

$\therefore$ Fare for next $(x-1)$ km $= (x-1) \times 7$

$= 7(x-1)$

Now, by given condition,

$$y = 10 + 7(x-1)$$

$$= 10 + 7x - 7$$

$\Rightarrow$ $= 7x + 3$ ...(1)

(ii) If man covers 16 km, i.e., $x = 16$, then from eq.(1), we get

$$y = (7 \times 16) + 3 = ₹115$$

$\therefore$ Amount paid back by auto driver

$$= ₹(120 - 115) = ₹5$$