CHAPTER 12

Heron's Formula

1. OBJECTIVE QUESTIONS

1. The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be
(a) $24\ \text{cm}^2$ (b) $40\ \text{cm}^2$
(c) $48\ \text{cm}^2$ (d) $80\ \text{cm}^2$

Ans : (a) $24\ \text{cm}^2$

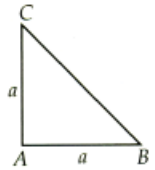
$$\text{Third side} = \sqrt{(\text{Hypotenuse})^2 - (\text{Base})^2}$$
$$= \sqrt{100 - 64} = 6\ \text{cm}$$
$$s = \frac{10+8+6}{2}\ \text{cm} = 12\ \text{cm}$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{12(12-10)(12-8)(12-6)}$$
$$= \sqrt{12 \times 2 \times 4 \times 6}\ \text{cm}^2$$
$$= 24\ \text{cm}^2$$

2. Each of equal sides of isosceles right triangle is 20 cm. What is the semi perimeter of the triangle?
(a) $20 + 10\sqrt{3}$ cm (b) $20\sqrt{2}$ cm
(c) $20 + 10\sqrt{2}$ cm (d) $40 + 20\sqrt{2}$ cm

Ans : (c) $20 + 10\sqrt{2}$ cm

Let ABC be an isosceles right triangle in which

$$AB = AC = 20\ \text{cm}$$
$$\text{Hypotenuse } BC = \sqrt{AB^2 + AC^2}$$
$$= \sqrt{a^2 + a^2}$$
$$= a\sqrt{2} = 20\sqrt{2}\ \text{cm}.$$
$$s = \frac{20 + 20 + 20\sqrt{2}}{2}$$
$$= \frac{40 + 20\sqrt{2}}{2} = 20 + 10\sqrt{2}$$



3. Area of the triangle whose sides are 13 cm, 9 cm and 6 cm is
(a) $23.6\ \text{cm}^2$ (b) $26.3\ \text{cm}^2$
(c) $36.34\ \text{cm}^2$ (d) $23.66\ \text{cm}^2$

Ans : (d) $23.66\ \text{cm}^2$

The sides of the triangle are $a = 13\ \text{cm}$, $b = 9\ \text{cm}$ and $c = 6\ \text{cm}$.

$$s = \frac{a+b+c}{2} = \frac{13+9+6}{2} = 14\ \text{cm}$$

Hence, area of the triangle
$$= \sqrt{s(s-a)(s-b)(s-c)} \quad \text{[Heron's formula]}$$
$$= \sqrt{14(14-13)(14-9)(14-6)}\ \text{cm}^2$$
$$= \sqrt{14 \times 1 \times 5 \times 8}\ \text{cm}^2$$
$$= \sqrt{560}\ \text{cm}^2 = 23.66\ \text{cm}^2$$

4. The value of semi-perimeter of an equilateral triangle having area $4\sqrt{3}\ \text{cm}^2$ is
(a) 8 cm (b) 36 cm
(c) 6 cm (d) 6 cm

Ans : (c) 6 cm

Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$$4\sqrt{3} = \frac{\sqrt{3}}{4}a^2$$
$$a^2 = 16$$
$$a = 4\ \text{cm}$$
$$\text{Semi-perimeter} = \frac{4+4+4}{2} = \frac{12}{2} = 6\ \text{cm}$$

5. The lengths of the three sides of a triangular field are 40 m, 24 m and 32 m respectively. The area of the triangle is
(a) $378\ \text{m}^2$ (b) $384\ \text{m}^2$
(c) $789\ \text{m}^2$ (d) $196\ \text{m}^2$

Ans : (b) $384\ \text{m}^2$

Here, $a = 40\ \text{m}$
$b = 24\ \text{m}$
and $c = 32\ \text{m}$
$$s = \frac{1}{2}(40 + 24 + 32) = 48\ \text{m}$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{48 \times 8 \times 24 \times 16} = 348\ \text{m}^2$$

6. If sides of a triangle are a, b and c then the value of semi-perimeter is
(a) $a^2 + b^2 + c^2$ (b) abc
(c) $a + b + c$ (d) $\sqrt{abc}$

Ans : (c) $a + b + c$

7. The sides of a triangular field are 33 m, 44 m and 55 m. the cost of levelling the field at the rate of ₹ 1.20 per m^2 is
(a) ₹ 871.20 (b) ₹ 1480
(c) ₹ 1560.20 (d) ₹ 980.60

Ans : (a) ₹ 1480

$$s = \frac{33+44+55}{2} = 66\,\text{m}$$

$$\text{Area of triangle} = \sqrt{66(66-33)(66-44)(66-55)}$$
$$= \sqrt{66\times 33\times 22\times 11}$$
$$= 726\,\text{m}^2$$
$$\text{Cost of levelling} = ₹\,(726\times 1.20) = ₹\,871.20$$

8. The sides of a triangle are 25 cm, 17 cm and 12 cm. The length of the altitude on the longest side is equal to

(a) 7.5 cm (b) 7.2 cm

(c) 8.2 cm (d) 9.8 cm

Ans : (b) 7.2 cm

$$s = \frac{25+17+12}{2} = 27\,\text{cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area of triangle} = \sqrt{27(27-25)(27-17)(27-12)}$$
$$= \sqrt{27\times 2\times 10\times 15}$$
$$= 90\,\text{cm}^2$$
$$\text{Also, area of triangle} = \frac{1}{2}\times 25\times h = 90$$
$$h = 90\times\frac{2}{25} = 7.2\,\text{cm}$$

9. An isosceles right-angled triangle has an area 8 cm^2. The value of perimeter of triangle is

(a) $4(2+\sqrt{2})$ cm (b) $4+\sqrt{2}$ cm

(c) $4+\sqrt{3}$ cm (d) $4\sqrt{2}+\sqrt{6}$ cm

Ans : (a) $4(2+\sqrt{2})$ cm

Let each equal sides of isosceles right angled triangle be a cm.

$$\frac{1}{2}a^2 = 8$$
$$a^2 = 16$$
$$a = 4$$
$$\text{Hypotenuse} = \sqrt{2a} = 4\sqrt{2}\ \text{cm}.$$
$$\text{Perimeter} = a+b+c$$
$$= 4+4+4\sqrt{2}$$
$$= 8+4\sqrt{2} = 4(2+\sqrt{2})\,\text{cm}$$

10. O is a point inside the parallelogram $ABCD$. Then the area of ΔAOB + area of ΔCOD is

(a) $1/3\times$ (area of ΔAOD + area of ΔBOC)

(b) area of ΔAOD + area of ΔBOC

(c) $2\times$ (area of ΔAOD + area of ΔBOC)

(d) $2/3\times$ (area of ΔAOD + area of ΔBOC)

Ans : (b) area of ΔAOD + area of ΔBOC

Since we know that area of

$$\Delta AOB = \text{area of } \Delta COD$$
$$= \text{area of } \Delta AOD$$
$$= \text{area of } \Delta BOC$$

Thus, area of ΔAOB + area of ΔCOD

$$= \text{area of } \Delta AOD + \text{area of } \Delta BOC$$

11. The perimeter of a right angled triangle is 450 m. If its sides are in the ratio 5 : 12 : 13, then area of the triangle is

(a) 9000 m^2 (b) 8765 m^2

(c) 6750 m^2 (d) 11750 m^2

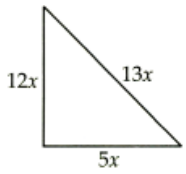
Ans : (c) 6750 m^2

Let the three sides be $5x$, $12x$ and $13x$

$$5x+12x+13x = 450$$
$$30x = 450$$
$$x = 15\,\text{m}$$
$$\text{Sides are } 5\times 15 = 75\text{ cm},$$
$$12\times 15 = 195\,\text{cm}$$
$$s = \frac{75+180+195}{2}$$
$$= \frac{450}{2} = 225\,\text{cm}$$

Area of triangle

$$= \sqrt{225(225-75)(225-180)(225-195)}$$
$$= \sqrt{225\times 150\times 45\times 30}$$
$$= 15\times 30\times 5\times 3 = 6750\,\text{m}^2$$



12. Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm.

(a) 56 cm^2 (b) 50 cm^2

(c) 52 cm^2 (d) 54 cm^2

Ans : (d) 54 cm^2

$$s = \frac{9+12+15}{2} = \frac{36}{2} = 18\,\text{cm}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{18(18-9)(18-12)(18-15)}$$
$$= \sqrt{18\times 9\times 6\times 3}$$
$$= \sqrt{9\times 2\times 9\times 3\times 2\times 3}$$
$$= 9\times 3\times 2 = 54\,\text{cm}^2$$

13. If the area of an equilateral triangle is $9\sqrt{3}$ cm^2, then the semi-perimeter of the triangle is

(a) 9 cm (b) 24 cm

(c) 12 cm (d) 10 cm

Ans : (a) 9 cm

$$\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4}a^2$$
$$\frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}$$
$$a^2 = 36$$
$$a = 6\,\text{cm}$$
$$\text{Semi-perimeter} = \frac{6+6+6}{2} = \frac{18}{2}$$

$= 9\,\text{cm}$

14. The percentage increase in the area of a triangle, if its each side is quadrupled, is equal to

(a) 1500% (b) 1200%

(c) 900% (d) 800%

Ans : (a) 1500%

$$s = \frac{1}{2}(a+b+c)$$

$$s' = \frac{1}{2}(4a+4b+4c) = 2(a+b+c) = 4s$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \text{ and}$$

$$\Delta' = \sqrt{s'(s'-4s)(s'-4a)(s'-4b)(s'-4c)}$$

$$= \sqrt{4s(4s-4a)(4s-4b)(4s-4c)}$$

$$= 16\sqrt{s(s-a)(s-b)(s-c)} = 16\Delta$$

Increase in the area of the triangle

$$= \Delta' - \Delta = 16\Delta - \Delta = 15\Delta$$

$$\text{Percentage increase} = \frac{15\Delta}{\Delta} \times 100 = 1500\%$$

15. If each side of the rhombus is 40 m and its longer diagonal is 48 m, then the area of rhombus is

(a) 1536 m^2 (b) 1636 m^2

(c) 1236 m^2 (d) 1336 m^2

Ans : (a) 1536 m^2

Here, each side of rhombus $= 40\,\text{m}$. One of the diagonal $= 48\,\text{m}$

$$a = 40\,\text{m},\ b = 40\,\text{m},\ c = 48\,\text{m}$$

$$s = \frac{a+b+c}{2} = \frac{40+40+48}{2}$$

$$= \frac{128}{2} = 64\,\text{m}$$

Area of triangle I

$$= \sqrt{64(64-40)(64-40)(64-48)}$$

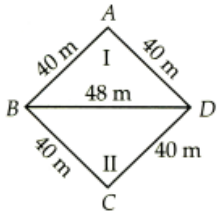
$$\sqrt{64(24)(24)(16)}$$

$$= 24 \times 16 \times 2 = 32 \times 24 = 768\,\text{m}^2$$

Area of triangle II $= 768\,\text{m}^2$.

So, area of rhombus

$$= 768\,\text{m}^2 + 768\,\text{m}^2 = 1536\,\text{m}^2$$



16. The length of the sides of a triangle are 4 cm, 6 cm and 8 cm. The length of perpendicular from the opposite vertex to the side whose length is 8 cm, is equal to

(a) $\frac{3}{4}\sqrt{15}\,\text{cm}$ (b) $\frac{\sqrt{3}}{5}\,\text{cm}$

(c) $\frac{3\sqrt{5}}{4}\,\text{cm}$ (d) $\frac{5\sqrt{3}}{4}\,\text{cm}$

Ans : (a) $\frac{3}{4}\sqrt{15}\,\text{cm}$

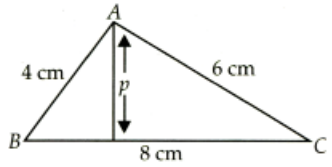
$$s = \frac{1}{2}(4+6+8)\,\text{cm} = 9\,\text{cm}$$

$$\text{Area} = \sqrt{9(9-4)(9-6)(9-8)}$$

$$= \sqrt{9 \times 5 \times 3 \times 1} = 3\sqrt{15}\,\text{cm}^2$$

$$\text{Also, area} = \frac{1}{2} \times 8 \times p$$

$$4p = 3\sqrt{15} \quad p = \frac{3\sqrt{15}}{4}\,\text{cm}$$



17. Find the percentage increase in the area of a triangle if its each side is doubled.

(a) 100% (b) 200%

(c) 400% (d) 300%

Ans : (d) 300%

Let a, b, c are the sides of the given triangle and s be its semi-perimeter

$$s = \frac{1}{2}(a+b+c) \qquad ...(1)$$

The sides of the new triangle are $2a$, $2b$ and $2c$.

Let s' be its semi-perimeter,

$$s = \frac{1}{2}(2a+2b+2c)$$

$$= a+b+c = 2s \qquad ...(2)$$

[using (1)]

Let $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ ...(3)

and $\Delta' = \sqrt{s'(s'-2a)(s'-2b(s'-2c)}$

$$= \sqrt{2s(2s-2a)(2s-2b)(s-c)}$$

$\Delta' = 4\Delta$ [using (3)]

Increase in the area of the triangle $= \Delta' - \Delta$

$$= 4\Delta - \Delta = 3\Delta$$

$$\%\text{ increase in area} = \left(\frac{3\Delta}{\Delta} \times 100\right)\% = 300\%$$

18. Using the area of the triangle whose sides are 42 cm, 34 cm and 20 cm in length, find the height corresponding to the shortest side.

(a) 14 cm (b) 15 cm

(c) 16 cm (d) 17 cm

Ans : (c) 16 cm

The sides of the triangle are

$$a = 42\,\text{cm}$$

$$b = 34\,\text{cm}$$

$$c = 20\,\text{cm}$$

$$s = \frac{a+b+c}{2} = \frac{42+34+20}{2}$$

$$= \frac{96}{2} = 48\,\text{cm}$$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{48 \times (48-42) \times (48-34) \times (48-20)}$$

$$= \sqrt{48 \times 6 \times 14 \times 28} = \sqrt{112896} = 336\,\text{cm}^2$$

The shortest side $= 20\,\text{cm}$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$336 = \frac{1}{2} \times 20 \times h$$

$$h = \frac{336}{10} = 33.6\,\text{cm}$$

19. The length of the sides forming right angle of a right angled triangle are $5x$ cm and $(3x-1)$ cm. If the area of the triangle is $60\,\text{cm}^2$, find its semi perimeter.

(a) 20 cm (b) 18 cm

(c) 19 cm (d) 16 cm

Ans : (a) 20 cm

Let ABC be a right angled triangle.

Let $AB = 5x$ cm

and $BC = (3x-1)\,\text{cm}$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$60 = \frac{1}{2} \times BC \times AB$$

$$60 = \frac{1}{2} \times (3x-1) \times 5x$$

$$120 = 5x(3x-1) \quad 24 = x(3x-1)$$

$$3x^2 - x - 24 = 0 \quad (x-3)\,((3x+8) = 0$$

$$x - 3 = 0 \text{ or } 3x + 8 = 0$$

$$x = 3 \qquad (\text{Since, } x \neq -8/3)$$

$$AB = 5x = 5 \times 3 = 15\,\text{cm}$$

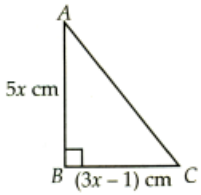
$$BC = (3x-1) = 3 \times 3 - 1 = 8\,\text{cm}$$

By Pythagoras theorem,

$$AC^2 = (15)^2 + (8)^2$$

$$= 225 + 64 = 289$$

$$AC = 17\,\text{cm}$$



20. A triangle and parallelogram have the same base and the same area. If the sides of the triangle are 34 cm, 42 cm and 20 cm, then the height of parallelogram having base 42 cm, is equal to

(a) 8 cm (b) 10 cm

(c) 11 cm (d) 12 cm

Ans : (a) 8 cm

$$s = \frac{42+34+20}{2} = \frac{96}{2} = 48\,\text{cm}$$

Area of the triangle

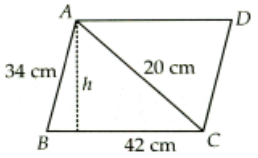
$$= \sqrt{48(48-42)(48-34)(48-20)}$$

$$= \sqrt{48 \times 6 \times 14 \times 28} = 336\,\text{cm}^2$$

$$\text{base} \times \text{height} = 336$$

$$42 \times h = 336$$

$$h = 8\,\text{cm}$$



21. Find the area of ΔABC in which $BC = 13$ cm, $AC = 14$ cm and $AB = 15$ cm.

(a) $84\ \text{cm}^2$ (b) $42\ \text{cm}^2$

(c) $82\ \text{cm}^2$ (d) $44\ \text{cm}^2$

Ans : (a) $84\ \text{cm}^2$

Here $a = 15\,\text{cm}$

$b = 14\,\text{cm}$

$c = 13\,\text{cm}$

$$s = \frac{a+b+c}{2} = \frac{15+14+13}{2} = \frac{42}{2}$$

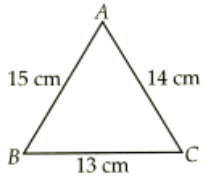
$$= 21\,\text{cm}$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-15)(21-14)(21-13)}$$

$$= \sqrt{21 \times 6 \times 7 \times 8} = \sqrt{7056} = 84\,\text{cm}^2$$

Thus, area of $\Delta ABC = 84\,\text{cm}^2$



22. A regular hexagon has a side 6 cm. Its perimeter and area are

(a) 35 cm, $8\sqrt{3}\ \text{cm}^2$ (b) 38 cm, $10\sqrt{2}\ \text{cm}^2$

(c) 40 cm, $11\sqrt{2}\ \text{cm}^2$ (d) 36 cm, $54\sqrt{3}\ \text{cm}^2$

Ans : (d) 36 cm, $54\sqrt{3}\ \text{cm}^2$

Side $= 6\,\text{cm}$

Hence, Perimeter of regular hexagon

$$= 6 \times 6 = 36\,\text{cm}$$

$ABCDEF$ is a regular hexagon. Join diagonals AD, BE and CF. The three diagonal divides the hexagonal in six. Congruent equilateral triangle with side 6 cm.

Area of one such triangle

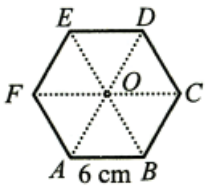
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{9(9-6)(9-6)(9-6)}$$

$$= \sqrt{9 \times 3 \times 3 \times 3} = 9\sqrt{3}$$

Hence, area of the regular hexagon

$$= 6 \times \text{Area of equilateral triangle } OAB$$



$= 6 \times 9\sqrt{3} = 54\sqrt{3}$ cm^2

23. If the perimeter of an equilateral triangle is 90 m, then its area is

(a) $15\sqrt{3}$ m^2 (b) $45\sqrt{3}$ m^2

(c) $225\sqrt{3}$ m^2 (d) $25\sqrt{3}$ m^2

Ans : (c) $225\sqrt{3}$ m^2

Let a be the side of given triangle.

Given, $3a = 90$

$a = 30$ m

$S = \frac{90}{2} = 45$ m

Area of triangle $= \sqrt{45(45-30)(45-30)(45-30)}$

$= \sqrt{45 \times 15 \times 15 \times 15}$

$= 15 \times 15\sqrt{3} = 225\sqrt{3}$ m^2

24. The area of an isosceles triangle having base 3 cm and length of one of the equal sides is 2 cm, is

(a) 1.98 cm^2 (b) 3.7 cm^2

(c) 2.5 cm^2 (d) 4.8 cm^2

Ans : (a) 1.98 cm^2

Here, $a = 3$ cm

$b = 2$ cm

$c = 2$ cm

$s = \frac{3+2+2}{2} = \frac{7}{2} = 3.5$ cm

Area of triangle $= \sqrt{3.5 \times (3.5-3) \times (3.5-2) \times (3.5-2)}$

$= \sqrt{3.5 \times 0.5 \times 1.5 \times 1.5}$

$= \sqrt{3.9375} = 1.98$ cm^2

25. An isosceles right triangle has area 8 cm^2. The length of its hypotenuse is

(a) $\sqrt{32}$ cm (b) $\sqrt{48}$ cm

(c) $\sqrt{24}$ cm (d) $\sqrt{16}$ cm

Ans : (a) $\sqrt{32}$ cm

Area of right triangle $= \frac{1}{2} \times b \times h$

Since its an isosceles right triangle

Hence, $b = h$

$\frac{1}{2} \times b^2 = 8$

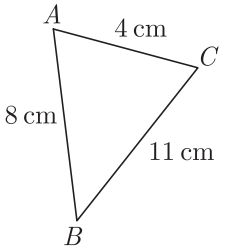
$b^2 = 16$

$b = 4$

Hypotenuse $= \sqrt{\text{base}^2 + \text{height}^2}$

$= \sqrt{4^2 + 4^2} = \sqrt{32}$ cm

26. In the given figure, the area of the ΔABC is



(a) 13.24 cm^2 (b) 12.28 cm^2

(c) 11.32 cm^2 (d) 15.37 cm^2

Ans : (b) 12.28 cm^2

Here, $a = 11$ cm

$b = 4$ cm

$c = 8$ cm

$s = \frac{11+4+8}{2} = \frac{23}{2} = 11.5$ cm

Area of triangle $= \sqrt{11.5 \times (11.5-11) \times (11.5-4) \times (11.5-8)}$

$= \sqrt{11.5 \times 0.5 \times 7.5 \times 3.5}$

$= \sqrt{150.94} = 12.28$ cm^2

27. In a triangle, the sides are 28 cm, 35 cm and 9 cm. Find the area of the triangle.

(a) $36\sqrt{7}$ cm^2 (b) $36\sqrt{6}$ cm^2

(c) $35\sqrt{5}$ cm^2 (d) $37\sqrt{7}$ cm^2

Ans : (b) $36\sqrt{6}$ cm^2

Here, $a = 28$ cm, $b = 35$ cm and $c = 9$ cm

$s = \frac{28+35+9}{2} = \frac{72}{2} = 36$ cm

Area $= \sqrt{36 \times (36-28) \times (36-35) \times (36-9)}$

$= \sqrt{36 \times 8 \times 1 \times 27} = 6 \times 6\sqrt{6} = 36\sqrt{6}$ cm^2

28. The base of right angled triangle measures 6 cm and its hypotenuse is 10 cm. The area of the triangle is

(a) 24 cm^2 (b) 12 cm^2

(c) 8 cm^2 (d) 16 cm^2

Ans : (a) 24 cm^2

$AC^2 = AB^2 + BC^2$

$10^2 = AB^2 + 6^2$

$AB^2 = 100 - 36 = 64$

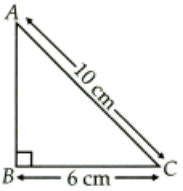
$AB = 8$ cm

$s = \frac{10+6+8}{2} = 12$ cm

Area of $\Delta ABC = \sqrt{12(12-10)(12-6)(12-8)}$

$= \sqrt{12 \times 2 \times 6 \times 4}$

$= 24$ cm^2



29. The base of a right angled triangle measures 48 cm and height is 50 cm. The area (approx). of the triangle is

(a) 1823 cm^2 (b) 1200 cm^2

(c) 1000 cm^2 (d) 1250 cm^2

Ans : (b) 1200 cm^2

Let ΔABC,

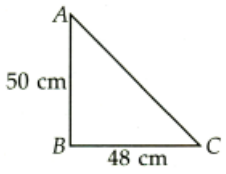
$$AC^2 = AB^2 + BC^2$$
$$= (50)^2 + (48)^2 = 2500 + 2304$$
$$= 4804$$
$$AC = 69.31 \text{ cm}$$

Now, $s = \frac{50 + 48 + 69.31}{2}$

$$= \frac{167.31}{2} = 83.65 \text{ cm}$$

Area of triangle

$$= \sqrt{83.65(83.65 - 48)(83.65 - 50)(83.65 - 69.31)}$$
$$= \sqrt{83.65 \times 35.65 \times 33.65 \times 14.34}$$
$$= 1200 \text{ cm}^2 \text{(Approx).}$$



30. The perimeter of the isosceles triangle is 60 cm and its base is 15 cm. The area of the triangle is (Take $\sqrt{2} = 1.414$)

(a) 160.75 cm^2 (b) 159.07 cm^2

(c) 156.75 cm^2 (d) 195.72 cm^2

Ans : (b) 159.07 cm^2

Perimeter of isosceles triangle $= 60$ cm

i.e., $x + x + 15 = 60$

$$2x = 45$$
$$x = 22.5 \text{ cm}$$

Now, $s = \frac{22.5 + 22.5 + 15}{2} = \frac{60}{2}$

$$= 30 \text{ cm}$$

Area of triangle

$$= \sqrt{30(30 - 22.5)(30 - 22.5)(30 - 15)}$$
$$= \sqrt{30 \times 7.5 \times 7.5 \times 15}$$
$$= 7.5 \times 15\sqrt{2}$$
$$= 7.5 \times 15 \times 1.414 = 159.07 \text{ cm}^2$$

31. The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Its area is

(a) $1500\sqrt{2}$ cm^2 (b) $1500\sqrt{3}$ cm^2

(c) $1425\sqrt{2}$ cm^2 (d) 1500 cm^2

Ans : (b) $1500\sqrt{3}$ cm^2

Suppose that the sides, in metres, are $3x$, $5x$ and $7x$. Then, we know that,

$$3x + 5x + 7x = 300$$

(Perimeter of the triangle)

Therefore, $15x = 300$

Which gives $x = 20$

So the sides of the triangle are 3×20 m, 5×20 m and 7×20 m

i.e., 60 m, 100 m, and 140 m

We have,

$$s = \frac{60 + 100 + 140}{2} \text{ m} = 150 \text{ m},$$

and area will be

$$= \sqrt{150(150 - 60)(150 - 100)(150 - 140)} \text{ m}^2$$
$$= \sqrt{150 \times 90 \times 50 \times 10} \text{ m}^2$$
$$= 1500\sqrt{3} \text{ m}^2$$

32. The length of two adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm long. Area of the parallelogram is

(a) $13\sqrt{10}$ cm^2 (b) $23\sqrt{5}$ cm^2

(c) $10\sqrt{5}$ cm^2 (d) $10\sqrt{3}$ cm^2

Ans : (d) $10\sqrt{3}$ cm^2

Area of the parallelogram, $ABCD$

$$= 2 \times (\text{Area of } \Delta ABC)$$

In ΔABC, $AB = 5$ cm, $BC = 3.5$ cm

and $AC = 6.5$ cm

So its perimeter, $s = \frac{5 + 3.5 + 6.5}{2} = 7.5$ cm

Since, area of ΔABC (By Heron's formula)

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{7.5 \times (7.5 - 5) \times (7.5 - 3.5) \times (7.5 - 6.5)}$$
$$= \sqrt{7.5 \times 2.5 \times 4 \times 1} \text{ Sq. cm}$$
$$= \sqrt{75} \text{ sq. cm} = 5\sqrt{3} \text{ sq. cm}$$

Area of parallelogram

$$= 2 \times 5\sqrt{3} \text{ sq.cm} = 10\sqrt{3} \text{ cm}^2$$

33. A triangular park ABC has sides 120 m, 80 m and 50 m. A gardener has to put a fence all around it and also plant grass inside. Area of garden and cost of fencing the garden with barbed wire at the rate of ₹ 20 per metre leaving a space 3 m wide for a gate on one side are

(a) $375\sqrt{15}$ m^2, ₹ 4940 (b) $357\sqrt{10}$ m^2, ₹ 9440

(c) $573\sqrt{8}$ m^2, ₹ 4944 (d) $683\sqrt{10}$ m^2, ₹ 5490

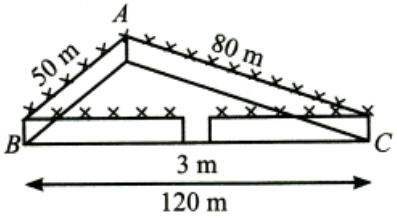
Ans : (a) $375\sqrt{15}$ m^2, ₹ 4940

For area of the park, we have

$$2s = 50 \text{ m} + 80 \text{ m} + 120 \text{ m} = 250 \text{ m}$$
$$s = 125 \text{ m}$$

i.e., $s - a = (125 - 120) \text{ m} = 5 \text{ m}$

$$s - b = (125 - 80) \text{ m} = 45 \text{ m}$$



$s - c = (125 - 80)\,\text{m} = 75\,\text{m}$

Therefore, area of the park

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}\ \text{m}^2$

$= 375\sqrt{15}\ \text{m}^2$

Also, perimeter of the park

$= AB + BC + CA = 250\,\text{m}$

Therefore, length of the wire needed for fencing

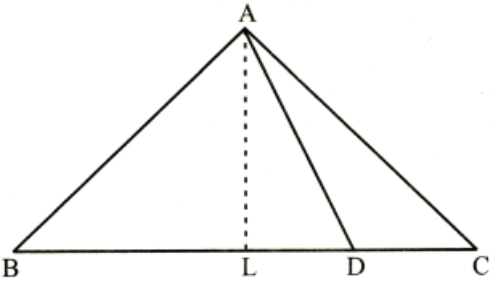
$= 250\,\text{m} - 3\,\text{m}$ (to be left for gate)

$= 247\,\text{m}$

And so the cost of fencing

$=₹\ 20 \times 247 =₹\ 4940$

34. AD is a median of a triangle ABC. If area of triangle $ADC = 15\,\text{cm}^2$, then ar (ΔABC) is



(a) $15\,\text{cm}^2$ (b) $22.5\,\text{cm}^2$

(c) $30\,\text{cm}^2$ (d) $37.5\,\text{cm}^2$

Ans : (c) $30\,\text{cm}^2$

AD is median of ΔABC.

area of ΔABD = area of ΔADC

$= \frac{1}{2}$ area of ΔABC ...(1)

Now, area of $\Delta ADC = 15\,\text{cm}^2$ (given)

Hence, From (1), $15 = \frac{1}{2}$ area of ΔABC

$30 =$ area of (ΔABC)

Hence,area of $\Delta ABC = 30\,\text{cm}^2$

35. From a point within an equilateral triangle, perpendiculars are drawn to its sides. The length of these perpendiculars are 6 m, 7 m and 8 m. The area of the triangle is

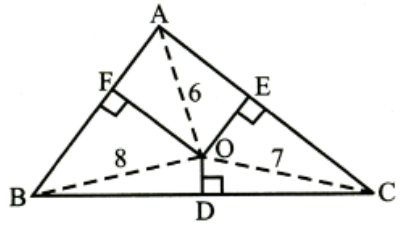
(a) 160 sq. m (b) $147\sqrt{3}$ sq. m

(c) $210\sqrt{3}$ sq. m (d) $27\sqrt{3}$ sq.m

Ans : (b) $147\sqrt{3}$ sq. m

Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2$

and, also



Area of ΔABC

= area of ΔBOC + area of ΔAOC + area of ΔAOB

$= \frac{1}{2} \times OD \times a + \frac{1}{2} \times OE \times a + \frac{1}{2} \times OF \times a$

$= a[6 + 7 + 8] = [21]$

$\frac{\sqrt{3}}{4}a^2 = \frac{a}{2}(21)$

$a = \frac{42}{\sqrt{3}} = 14\sqrt{3}$

Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times (14\sqrt{3})^2 = 147\sqrt{3}\ \text{m}^2$

2. FILL IN THE BLANK

1. Area of a triangle with the length of sides a, b, c is given by $\sqrt{s(s-a)(s-b)(s-c)}$ by ..... formula.

Ans : Heron's

2. Perimeter of an equilateral triangle is always equal to ..... times of length of sides.

Ans : three

3. If area of an equilateral triangle is $100\sqrt{3}\ \text{cm}^2$ then perimeter of this triangle will be .....

Ans : 60 cm

4. If each side of a scalene triangle is halved then its area will reduced by ..... percentage.

Ans : 75%

5. If height of a triangle is doubled and base in tripled then its area become ..... times.

Ans : six

6. If height of a triangle is halved then its area will become ..... of original area.

Ans : half

7. Area of an equilateral triangle is always a/an ..... number [Given that length of each side is rational]

Ans : irrational

8. If base of a triangle is doubled then its area will be ..... times of original area.

Ans : two

9. Area of a triangle with perimeter 42 cm and length of two sides 18 cm and 10 cm is given by .....

Ans : $21\sqrt{11}\ \text{cm}^2$

10. If length of hypotenuse of an isosceles right angled triangle is $10\sqrt{2}$ cm then its perimeter will be .....

Ans : $10\sqrt{2}(\sqrt{2}+1)$ cm.

3. TRUE/FALSE

1. If the adjacent angles of a rhombus of side 10 cm are 120° and 60°, then its area is $25\sqrt{3}$ cm^2.

Ans : False

2. Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using the Heron's formula.

Ans : True

3. Area of a quadrilateral $ABCD$ in which $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm is 15 cm^2

Ans : False

4. If P is any point in the interior of a rectangle $ABCD$, then Area (ΔPAB) + Area (ΔPCD) = Area (ΔPBC) + Area (ΔPDA).

Ans : True

5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Its area is 8000 cm^2.

Ans : False

6. Area of a triangle whose sides are 13 cm, 14 cm and 15 cm is 84 cm^2.

Ans : True

7. Area of the triangle whose two sides are 8 m and 11 m and perimeter is 32 m, is $8\sqrt{30}$ m^2.

Ans : True

8. The sides of a quadrilateral taken in order are 5 m, 12 m, 14 m and 15 m. If the angle between the first two sides be 90°, its area 114 m^2.

Ans : True

9. Heron's formula cannot be use to calculate area of quadrilaterals.

Ans : False

4. MATCHING QUESTIONS

DIRECTION : In the section, each question has two matching lists. Choices for the correct combination of elements from Column-I and Column-II are given as options (a), (b), (c) and (d) out of which one is correct.

1. Match the following lists:

Column-I		**Column-II**	
(P)	The length of three sides of triangle are 26 cm, 28 cm and 30 cm. The height (in cm) corresponding to base 28 cm is	(1)	9
(Q)	The area of an equilateral triangle is $3\sqrt{3}$ cm^2. The semi-perimeter of the triangle (in cm) is	(2)	13.63
(R)	If in ΔABC, $AB = 6$ cm, $BC = 7$ cm and $AC = 5$ cm, then the value of s (in cm) is	(3)	24
(S)	Let the base of an isosceles triangle be 5 cm and each of the equal sides be 6 cm. Then, its area (in cm^2) is	(4)	$3\sqrt{3}$

	P	**Q**	**R**	**S**
(a)	3	1	4	2
(b)	3	4	1	2
(c)	1	2	3	4
(d)	1	3	2	4

Ans : (b) P–3, Q–4, R–1, S–2

(P) $a = 26$ cm, $b = 28$ cm, $c = 30$ cm

$$s = \frac{1}{2}(26 + 28 + 30) = 42 \text{ cm}$$

$$\text{Area} = \sqrt{42(42-26)(42-28)(42-30)}$$
$$= \sqrt{42 \times 16 \times 14 \times 12} = 336 \text{ cm}^2$$

Now, $\frac{1}{2} \times 28 \times h = 336$

$$h = \frac{336}{14} = 24 \text{ cm}$$

(Q) $\frac{\sqrt{3}}{4}a^2 = 3\sqrt{3}$

$$a^2 = 12$$
$$a = 2\sqrt{3} \text{ cm}$$

Hence,Semi-perimeter$= \frac{3 \times 2\sqrt{3}}{2} = 3\sqrt{3}$ cm

(R) Here, $a = 7$ cm, $b = 5$ cm, $c = 6$ cm

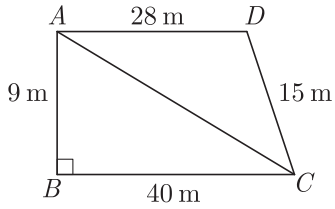
$$s = \frac{a+b+c}{2} = \frac{7+5+6}{2} = 9 \text{ cm}$$

(S) Here, $a = 5$ cm, $b = 6$ cm, $c = 6$ cm

$$s = \frac{6+6+5}{2} = \frac{17}{2} = 8.5 \text{ cm}$$

$$\text{Area} = \sqrt{8.5(8.5-6)(8.5-6)(8.5-5)}$$
$$= \sqrt{8.5 \times 2.5 \times 2.5 \times 3.5}$$
$$= \sqrt{185.93} = 13.63 \text{ cm}^2$$

2. Match the lists. For the given figure.



	Column-I		Column-II
(P)	Area of ΔABC is	(1)	41 m
(Q)	Area of ΔACD is	(2)	126 m^2
(R)	Total area of $ABCD$ is	(3)	306 m^2
(S)	AC is	(4)	180 m^2

	P	Q	R	S
(a)	4	2	3	1
(b)	4	3	2	1
(c)	1	2	3	4
(d)	1	3	2	4

Ans : (a) P–4, Q–2, R–3, S–1

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 40 \times 9 = 180\,m^2$$

Since, ΔABC is right angled triangle

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$= (9)^2 + (40)^2$$
$$= 81 + 1600 = 1681$$
$$AC = \sqrt{1681} = 41\,m$$

Now, in ΔACD, $a = 28m$, $b = 15$ m, $c = 41$ m

$$s = \frac{28+15+41}{2} = \frac{84}{2} = 42 \text{ m}$$

Area of ΔACD

$$= \sqrt{42 \times (42-28) \times (42-15) \times (42-41)}$$
$$= \sqrt{42 \times 14 \times 27 \times 1} = 14 \times 9 = 126\,m^2$$

Area of quadrilateral $ABCD$

$$= \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$
$$= 180 + 126 = 306\,m^2$$

3. Match the following lists:

	Column-I		Column-II
(P)	The sides of a triangle are 35 cm, 54 cm and 61 cm. The length of the longest altitude is	(1)	612
(Q)	If the side of a rhombus is 10 cm and one diagonal is 16 cm, then area of the rhombus is	(2)	96
(R)	The cost of levelling the ground in the form of triangle having sides 51 m, 37 m and 20 m at the rate of ₹ 2 per m^2 is	(3)	$24\sqrt{5}$

Ans : P–3, Q–2, R–1

(P) $\text{Area of triangle} = 420\sqrt{5}\ cm^2$

Hence, $\text{Longest altitude} = \frac{2 \times 420\sqrt{5}}{35}$ cm

(Q) $\text{Area of rhombus} = 2 \times \text{area of } \Delta$

Now, $\text{area of triangle} = \sqrt{18(8)(2)}$

$$= 2 \times 2 \times 2 \times 2 \times 3$$
$$= 48$$

Hence, $\text{area of rhombus} = 2 \times 48 = 96\,cm^2$

(R) $\text{Area of the triangle} = 306\,m^2$

cost of the rate of ₹2 per $m^2 = 306 \times 2 = 612$

4. Match the following lists:

	Column-I		Column-II
(P)	The area of a triangle with base 4 cm and height 6 cm is	(1)	$\frac{5}{4}\sqrt{11}\ cm^2$
(Q)	If the perimeter of the isosceles triangle is 11 cm and the base is 5 cm then the area of the isosceles triangle is	(2)	$16\sqrt{3}\ cm^2$
(R)	The area of the equilateral triangle, whose each side is 8 cm, is	(3)	$12\,cm^2$

Ans : P–3, Q–1, R–2

(P) $\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 4 \times 6 = 12\,cm^2$$

(Q) Perimeter of isosceles triangle $= 11$

$$x + x + 5 = 11$$
$$2x = 6$$
$$x = 3$$

$$\text{Area of isosceles triangle} = \frac{5}{4}\sqrt{36-25}\ cm^2$$
$$= \frac{5}{4}\sqrt{11}\ cm^2$$

(R) Area of equilateral triangle

$$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(8)(8)$$
$$= 16\sqrt{3}\ cm^2$$

5. ASSERTION AND REASON

DIRECTION : In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

1. **Assertion :** The sides of a triangle are 3 cm, 4 cm and 5 cm. Its area is $6\,\text{cm}^2$.
Reason : If $2\,s = (a+b+c)$, where a, b, c are the sides of a triangle, then area $= \sqrt{(s-a)(s-b)(s-c)}$.
Ans : (c) Assertion is correct but Reason is incorrect.

$$s = \frac{a+b+c}{2}$$

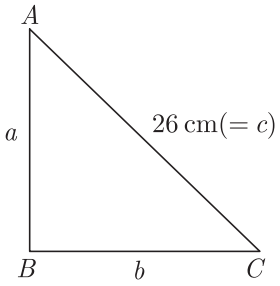
$$s = \frac{3+4+5}{2} = 6\,\text{cm}$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(6)(6-3)(6-4)(6-5)} \\ &= \sqrt{(6)(3)(2)(1)} = 6\,\text{cm}^2\end{aligned}$$

2. **Assertion :** The side of an equilateral triangle is 6 cm then the area of the triangle is 9 cm^2
Reason : All the sides of an equilateral triangle are equal.
Ans : (d) Assertion is false but reason is true.

$$\begin{aligned}s &= \frac{6+6+6}{2} = \frac{18}{2} = 9\,\text{cm} \\ \text{Area} &= \sqrt{9(9-6)(9-6)(9-6)} \\ &= \sqrt{9\times 3\times 3\times 3} = 9\sqrt{3}\ \text{cm}^2\end{aligned}$$

3. **Assertion :** The perimeter of a right angled triangle is 60 cm and its hypotenuse is 26 cm. The other sides of the triangle are 10 cm and 24 cm. Also, area of the triangle is 120 cm^2.
Reason : $(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$
Ans : (a) Both assertion and reason are true and reason is the correct explanation of assertion.



$$\begin{aligned}a+b+c &= 60 \\ a+b+26 &= 60 \\ a+b &= 34 \quad ...(1)\end{aligned}$$

Now, $26^2 = a^2+b^2$ ...(2)

Squaring (1) both sides, we get

$$\begin{aligned}(a+b)^2 &= (34)^2 \\ a^2+b^2+2ab &= 34\times 34 \\ (26)^2+2ab &= 1156 \quad [\text{From (2)}] \\ 2ab &= 1156-676 \\ 2ab &= 480 \\ ab &= 240 \quad ...(3)\end{aligned}$$

Now, $a+\frac{240}{a} = 34$ [From (1) and (3)]

$$\begin{aligned}a^2-24a-10a+240 &= 0 \\ a(a-24)-10(a-24) &= 0 \\ a &= 10,\ 24\end{aligned}$$

Now, other sides are 10 cm and 24 cm.

$$s = \frac{26+10+24}{2} = 30\,\text{cm}$$

Area of triangle

$$\begin{aligned}&= \sqrt{30(30-26)(30-10)(30-24)} \\ &= \sqrt{30\times 4\times 20\times 6} = 120\,\text{cm}^2\end{aligned}$$

4. **Assertion :** The side of an equilateral triangle is 6 cm then the height of the triangle is 9 cm.
Reason : The height of an equilateral triangle is $\frac{\sqrt{3}}{2}a$.
Ans : (d) Assertion is incorrect but Reason is correct.
The height of the triangle,

$$h = \frac{\sqrt{3}}{2}a$$

$$9 = \frac{\sqrt{3}}{2}a$$

$$\begin{aligned}a &= \frac{9\times 2}{\sqrt{3}} = \frac{18}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{18\sqrt{3}}{3} = 6\sqrt{3}\ \text{cm}\end{aligned}$$

5. **Assertion :** The height of the triangle is 18 cm and its area is $72\,\text{cm}^2$. Its base is 8 cm.
Reason : Area of a triangle $= \frac{1}{2}\times$ base $\times$ height.
Ans : (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

$$\text{Assertion : Area of } \Delta = \frac{1}{2}\times \text{base} \times \text{height}$$

$$72 = \frac{1}{2}\times 18\times b$$

$$b = \frac{72\times 2}{18} = 8\,\text{cm}$$

6. **Assertion :** The sides of a triangle are 3 cm, 4 cm and 5 cm. Its area is 6 cm^2.
Reason : If $2s = (a+b+c)$, where a, b, c are the sides of a triangle, then area

$$= \sqrt{(s-a)(s-b)(s-c)}$$

Ans : (c) Assertion is true but reason is false.

$$s = \frac{a+b+c}{2}$$

$$s = \frac{3+4+5}{2} = 6\,\text{cm}$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(6)(6-3)(6-4)(6-5)} \\ &= \sqrt{6(3)(2)(1)} = 6\,\text{cm}^2\end{aligned}$$

7. **Assertion :** If the area of an equilateral triangle is $81\sqrt{3}\ \text{cm}^2$, then the semi perimeter of triangle is 20 cm.
Reason : Semi perimeter of a triangle is

$s = \frac{a+b+c}{2}$, where a, b, c are sides of triangle.

Ans : (d) Assertion is false but reason is true.

Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$, where a is side of triangle

$$81\sqrt{3} = \frac{\sqrt{3}}{4}a^2$$
$$81 \times 4 = a^2$$
$$324 = a^2$$
$$a = 18\,\text{cm}$$
$$s = \frac{18+18+18}{2} = 27\,\text{cm}$$

8. **Assertion :** The sides of a triangle are in the ratio of 25 : 14 : 12 and its perimeter is 510 m. Then the greatest side is 250 cm.
Reason : Perimeter of a triangle $= a+b+c$, where a, b, c are sides of a triangle.

Ans : (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

$$510 = a+b+c$$
$$510 = 25x+14x+12x$$
$$510 = 51x$$
$$x = 10$$

Three sides of the triangle are,

$$25x = 25 \times 10 = 250\,\text{cm}$$
$$14x = 14 \times 10 = 140\,\text{cm}$$

and $12x = 12 \times 10 = 120\,\text{cm}$

9. **Assertion :** The sides of a triangle are in the ratio of 25 : 14 : 12 and its perimeter is 510 cm. Then the area of the triangle is 4449.08 cm^2.
Reason : Perimeter of a triangle $= a+b+c$, where a, b, c are sides of a triangle.

Ans : (a) Both assertion and reason are true and reason is the correct explanation of assertion.

$$510 = a+b+c$$
$$510 = 25x+14x+12x$$
$$510 = 51x$$
$$x = 10$$

Three side of the triangle are

$$25x = 25 \times 10 = 250\,\text{cm}$$
$$14x = 14 \times 10 = 140\,\text{cm and}$$
$$12x = 12 \times 10 = 120\,\text{cm}$$
$$s = \frac{250+140+120}{2} = 255\,\text{cm}$$
$$\text{Area} = \sqrt{255 \times 5 \times 115 \times 135}$$
$$= 4449.08\,\text{cm}^2$$