CLASS XII (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-5

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 36 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION-A

DIRECTION : (Q 1-Q 10) are multiple choice type questions. Select the correct option.

Q1. Let R be the relation in the set $\{1,2,3,4\}$ given by $R=\{(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$. Then, [1]

(a) R is reflexive and transitive but not symmetric
(b) R is reflexive and symmetric but not transitive
(c) R is symmetric and transitive but not reflexive
(d) R is an equivalence relation

Q2. The normal at the point (0, 1) on the curve $y=e^{2x}+x^2$ is [1]

(a) $x+y=0$ (b) $x+2y=2$
(c) $x+2y+1=0$ (d) $x-y+1=0$

Q3. The probability of obtaining an even prime number on each die when a pair of dice is rolled, is [1]

(a) zero (b) $\frac{1}{3}$
(c) $\frac{1}{12}$ (d) $\frac{1}{36}$

Q4. If $\vec{a}$ is a non-zero vector of magnitude $|\vec{a}|$ and λ is a non-zero scalar, then λa is unit vector, if [1]

(a) $\lambda=1$ (b) $\lambda=-1$
(c) $|\vec{a}|=|\lambda|$ (d) $|\vec{a}|=\frac{1}{|\lambda|}$

Q5. $\int_{-5}^{-5}|x+2|dx$ is equal to [1]

(a) 22 (b) 29
(c) 35 (d) 15

Q6. The number of arbitrary constants in the particular solution of differential equation of third order is [1]

(a) 3 (b) 2
(c) 1 (d) 0

Q7. The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue when $x = 15$ is [1]

(a) 116 (b) 96
(c) 90 (d) 126

Q8. $\int_0^2 x\sqrt{2-x}\,dx$ is equal to [1]

(a) $\frac{16\sqrt{2}}{15}$ (b) $\frac{3\sqrt{2}}{5}$
(c) $\frac{4\sqrt{3}}{5}$ (d) $\frac{6\sqrt{5}}{7}$

Q9. For the function $f(x) = xe^x$, the point [1]

(a) $x = 0$ is a maximum (b) $x = 0$ is a minimum
(c) $x = -1$ is a maximum (d) $x = -1$ is a minimum

Q10. $\int_0^2 \{x\}\,dx$ is equal to (where $\{x\}$ is fraction part of x) [1]

(a) 2 (b) 1
(c) 5 (d) 4

DIRECTION : (Q 11-Q 15) fill in the blanks

Q11. A feasible solution which leads to an optimal value of the objective function is called .... [1]

Q12. The range of $\cos^{-1}x$ is .... [1]

Q13. Every differentiable function is continuous. But a continuous function may or may not be ....[1]

OR

Let $f:[a,b] \to R$ be a continuous function on $[a,b]$ and differential function in $[a,b]$. By mean value theorem, there exists atleast one c in $[a,b]$ such that $f'(c) =$ .... [1]

Q14. If A and B are square matrices such that $AB = BA$, then $(A+B)^2 =$ .... [1]

OR

Transpose of a column matrix is a ....

Q15. $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx +$ .... [1]

DIRECTION : (Q 16-Q 20) Answer the following questions.

Q16. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots\infty$, then prove that $\frac{d^2y}{dx^2} - y = 0$. [1]

Q17. If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$, then find $|3AB|$. [1]

Q18. Find the direction cosines of the line passing through the two points (–2, 4, –5) and (1, 2, 3). [1]

OR

Find the distance of the point whose positive vector is $(2\hat{i} + \hat{j} - \hat{k})$ from the plane $\vec{r}(\hat{i} - 2\hat{j} + 4\hat{k}) = 9$.

Q19. Evaluate $\int_0^1 3^{x-[x]}dx$. [1]

Q20. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'number is even' and B be the event, 'number is red'. Are A and B are independent ? [1]

SECTION B

Q21. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B, respectively, find the position vector of a point C on BA produced such that $BC = 1.5BA$. [2]

Q22. Show that the function $f(x)$ given by $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$. [2]

OR

Differentiate $(\log \sin x)$ with respect to $\sqrt{\cos x}$.

Q23. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R. [2]

Q24. A fair die is rolled. Consider the following events $A = \{2,4,6\}$, $B = \{4,5\}$ and $C = \{3,4,5,6\}$. Find

(i) $P\left(\frac{A \cup B}{C}\right)$,

(ii) $P\left(\frac{A \cap B}{C}\right)$. [2]

Q25. Show that the determinant value of a skew-symmetric matrix of odd order is always zero. [2]

OR

Without expanding, show that

$$\Delta = \begin{vmatrix} \text{cosec}^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \text{cosec}^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$$

Q26. Find the minimum value of n for which $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}$, $n \in N$. [2]

SECTION C

Q27. Find the equation of a curve passing through the point (0, 1), if the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x-coordinate (abscissa) and the product of the x-coordinate and y-coordinate (ordinate) of that point. [4]

Q28. Evaluate $\int \frac{1+x^2}{1+x^4}dx$. [4]

OR

Evaluate $\int x \cdot (\log x)^2 dx$.

Q29. A can hit target 4 times out of 5 times, B can hit target 3 times out of 4 times and C can hit target 2 times out of 3 times.

They fire simultaneously. Find the probability that
(i) any two out of A, B and C will hit the target.
(ii) none of them will hit the target.

OR

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that, a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that a student knows the answer given that he answered it correctly ?

Q30. Let $\vec{a} = 2\hat{i} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ be three vectors. Find a vector $\vec{r}$ which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$. [4]

Q31. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is almost half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by almost 600 units. If the company makes profit of ₹ 12 and ₹ 16 per doll, respectively on dolls A and B, then how many of each should be produced weekly in order to maximise the profit ? [4]

OR

If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a} \neq \vec{0}$, then prove that $\vec{b} = \vec{c}$.

Q32. Show that $f : R - (-1) \rightarrow R - \{1\}$ given by $f(x) = \frac{x}{x+1}$ is invertible. Also, find f^{-1}. [4]

SECTION D

Q33. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$ and $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin. [6]

OR

If the length of three sides of a trapezium other than base are equal to 10 cm, find the area of the trapezium when it is maximum.

Q34. Find the image of the point (1, 6, 3) on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, write the equation of the line joining the given point and its image and find the length of segment joining the given point and its image. [6]

OR

Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ Also, find the length of the perpendicular.

Q35. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$. [6]

Q36. Solve the following system of equations by matrix method, where $x \neq 0$, $y \neq 0$ and $z \neq 0$.

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10,$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$

and $$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

[6]