# CLASS XII (2019-20)
# MATHEMATICS (041)
# SAMPLE PAPER-4

**Time : 3 Hours** **Maximum Marks : 80**

**General Instructions :**

(i) All questions are compulsory.
(ii) The questions paper consists of 36 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

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## SECTION-A

**DIRECTION : (Q 1-Q 10) are multiple choice type questions. Select the correct option.**

Q1. The domain of the function $\cos^{-1}(2x-1)$ is [1]

(a) [0, 1] (b) [–1, 1]
(c) (–1, 1) (d) $[0, \pi]$

Q2. If $2x+3y=\sin x$, then $\frac{dy}{dx}$ is equal to [1]

(a) $\frac{\cos x+2}{3}$ (b) $\frac{\cos x-2}{3}$
(c) $\cos x+2$ (d) None of these

Q3. The value of $\int \frac{1}{x-\sqrt{x}}\,dx$ is [1]

(a) $2\log\sqrt{x}+c$ (b) $\frac{x}{x-\sqrt{x}}+c$
(c) $2\log(\sqrt{x}-1)+c$ (d) $\log(\sqrt{x}-1)+c$

Q4. The function $f(x)=[x]$, where $[x]$ denotes the greatest integer function is continuous at [1]

(a) 4 (b) –2
(c) 1 (d) 1.5

Q5. $\int e^x(\cos x-\sin x)\,dx$ is equal to [1]

(a) $e^x\cos x+c$ (b) $e^x\sin x+c$
(c) $-e^x\cos x+c$ (d) $-e^x\sin x+c$

Q6. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=4$, then $|\vec{a}-\vec{b}|$ is equal to [1]

(a) $\sqrt{3}$ (b) $\sqrt{5}$
(c) $\sqrt{7}$ (d) $2\sqrt{2}$

Q7. $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ is equal to [1]

(a) $\pi$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Q8. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then $P(\frac{A}{B})$ is equal to [1]

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{1}{9}$

Q9. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is equal to [1]

(a) $\frac{\pi}{4}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Q10. $\int_0^{\pi} \frac{x}{1+\sin x} dx$ is equal to [1]

(a) $3\pi$ (b) $\pi$

(c) $2\pi$ (d) 0

**DIRECTION : (Q 11-Q 15) fill in the blanks**

Q11. The linear inequalities of equation or restrictions on the variables of a linear programming problem are called ........... [1]

**OR**

Variables of the objective function of the linear programming problem are ............

Q12. If $A$ and $B$ are square matrices of the same order, then $(AB)' =$ ............ [1]

**OR**

A square matrix whose diagonal element is unity and other elements are zero is called ............

Q13. Another name for the mean of a probability distribution is ............ [1]

Q14. The number of arbitrary constants in the general solution of a differential equation of order three is ............ [1]

Q15. If $f'(x)$ changes sign from negative to positive as $x$ increases through $c$, then $c$ is a point of ............ [1]

**DIRECTION : (Q 16-Q 20) Answer the following questions.**

Q16. If $X + \begin{bmatrix} 4 & 6 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 5 & -8 \end{bmatrix}$, then matrix $X$ is ? [1]

Q17. Find a vector in the direction of a vector $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, which has magnitude 8 units. [1]

**OR**

If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

Q18. Let $f: R \to R$, $f(x) = (x^2 - 3x + 2)$. Find $fof(x)$. [1]

Q19. If $x$ changes from 3 to 3.3, find the approximate change in $\log_e(1+x)$. [1]

Q20. Show that the function:
$f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima. [1]

## SECTION B

Q21. Show that the points $(a+5, a-4)$, $(a-2, a+3)$ and $(a, a)$ do not lie on a straight line for any value of $a$. [2]

Q22. If $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{5}$ and $P(A \cap B) = \frac{1}{7}$, find $P\left(\frac{\overline{A}}{\overline{B}}\right)$. [2]

Q23. Determine $f(0)$, so that the function $f(x)$ defined by $f(x) = \dfrac{(4^x - 1)^3}{\sin\frac{x}{4}\log\left(1+\frac{x^2}{3}\right)}$ becomes continuous at $x = 0$. [2]

**OR**

If $y = b\tan^{-1}\left(\frac{x}{a} + \tan^{-1}\frac{y}{x}\right)$, find $\frac{dy}{dx}$.

Q24. Solve for $x$ [2]

$$\cos(2\sin^{-1}x) = \frac{1}{9}, \quad x > 0$$

**OR**

Evaluate $\cos\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right]$.

Q25. Evaluate $\int \tan(x-\theta)\tan(x+\theta)\tan 2x \, dx$. [2]

Q26. Find the position vector of a point $R$ which divides the line joining the points $P(\hat{i} + 2\hat{j} - \hat{k})$ and $Q(-\hat{i} + \hat{j} + \hat{k})$ in the ratio $2:1$

(i) internally
(ii) externally [2]

## SECTION C

Q27. Find the shortest distance between lines [4]

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

and $$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

Q28. Find the particular solution of the differential equation $(1+e^{2x})\,dy + (1+y^2)e^x\,dx = 0$, given that $y = 1$ when $x = 0$. [4]

**OR**

Show that the differential equation that represents the family of all parabolas having their axis of symmetry coincident with the axis of $x$ is $yy_2 + y_1^2 = 0$.

Q29. Evaluate $\int \sqrt{3-4x-4x^2}\,dx$. [4]

**OR**

Evaluate $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$.

Q30. An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket.

The airline donate its 5% of total profit in welfare fund for poor girls. The airline reserves atleast 20 seats for executive class. However, atleast 4 times as many passengers prefer to travel by economy class, then by executive class. Determine how many tickets of each type must be sold in order to maximise profit for the airline ? What is the maximum profit? [4]

Q31. Find the mean and variance of number of tails when a coin is tossed thrice. [4]

**OR**

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15, respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Q32. Let $T$ be the set of all triangles in a plane. Let us define a relation $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2, T_1, T_2 \in T\}$. Show that $R$ is an equivalence relation. [4]

## SECTION D

Q33. Find the area of the region bounded by the parabola $x^2 = 4y$ and the line $x = 4y - 2$. [6]

Q34. Show that of all the rectangles with a given perimeter, the square has the largest area. [6]

Q35. Find the image of point (1, 0, 0) on the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$. [6]

**OR**

Find the equation of the plane that contains the point (1, –1, 2) and is perpendicular to both the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$. Hence, find the distance of point $P(-2,5,5)$ from the plane obtained above.

Q36. Show that $\Delta ABC$ is an isosceles triangle, if the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos B \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix} = 0$$

[6]

**OR**

If $A + B + C = \pi$, show that

$$\begin{vmatrix} \sin^2 A & \sin A\cos A & \cos^2 A \\ \sin^2 B & \sin B\cos B & \cos^2 B \\ \sin^2 C & \sin C\cos C & \cos^2 C \end{vmatrix} = \sin(A-B)\sin(B-C)\sin(C-A)$$