CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-9

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. Four rational numbers between 3 and 4 are: [1]

(a) $\frac{3}{5}, \frac{4}{5}, 1, \frac{6}{5}$ (b) $\frac{13}{5}, \frac{14}{5}, \frac{16}{5}, \frac{17}{5}$

(c) 3.1, 3.2, 4.1, 4.2 (d) 3.1, 3.2, 3.8, 3.9

Ans : (d) 3.1, 3.2, 3.8, 3.9

To find four rational numbers between 3 and 4.

$\frac{3\times 5}{5}$ and $\frac{4\times 5}{5}$

$\frac{15}{5}$ and $\frac{20}{5}$

Between $\frac{15}{5}$ and $\frac{20}{5}$ lies $\frac{16}{5}, \frac{17}{5}, \frac{18}{5}, \frac{19}{5}$

Now, from the given options (a) and (b) does not contain rational number between 3 and 5.
(c) has 4.1 and 4.2 that does not lie between 3 and 4.

2. In the method of factorisation of an algebraic expression, which of the following statement is false? [1]

(a) Taking out a common factor from two or more terms.
(b) Taking out a common factor from a group of terms.
(c) Using remainder theorem.
(d) Using standard identities.

Ans : (c) Using remainder theorem.

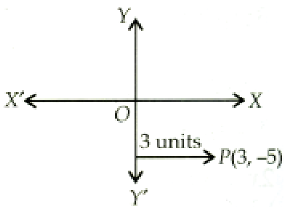
Remainder theorem is not used for factorisation of an algebraic expression.

3. If the coordinates of the point P are $(3, -5)$ then the perpendicular distance of P from the y-axis. [1]

(a) 4 (b) 5
(c) 3 (d) 2

Ans : (c) 3

Since, the abscissa is 3.



Perpendicular distance from the y-axis is 3 units.

4. The graph of $y = 6$ is a line [1]

(a) parallel to x-axis at a distance 6 units from the origin
(b) parallel to y-axis at a distance 6 units from the origin
(c) making an intercept 6 on the x-axis
(d) making an intercept 6 on both the axes

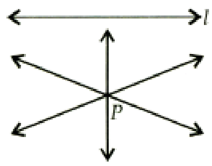
Ans : (a) parallel to x-axis at a distance 6 units from the origin

5. For every line l and for every point P (not on l), there does not exist a unique line through P [1]

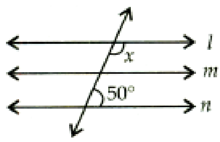
(a) Which is not parallel to l.
(b) Which is perpendicular to l.
(c) Which is coincident with l.
(d) None of these

Ans : (a) Which is not parallel to l.

There can be infinite lines that can be drawn through P not || to l but there exist a unique line through P which is parallel to l.



6. In figure, if $l \parallel m$, $m \parallel n$, then $x =$ [1]



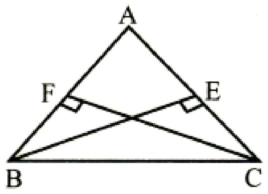
(a) $130°$ (b) $140°$
(c) $120°$ (d) $154°$

Ans : (a) $130°$

Since $l \parallel m$ and $m \parallel n$, then $l \parallel n$

$$x + 50° = 180° \quad \text{[Co-interior Angles]}$$
$$x = 130°$$

7. In the given figure if $BE = CF$, then [1]



(a) $\Delta ABE \cong \Delta ACF$ (b) $\Delta ABE \cong \Delta AFC$
(c) $\Delta ABE \cong \Delta CAF$ (d) $\Delta AEB \cong \Delta AFC$

Ans : (a) $\Delta ABE \cong \Delta ACF$

In triangle ABE and ACF,

$$BE = CF$$
$$\angle CFA = \angle BEA = 90^\circ$$

$\angle A$ is common.

Hence, $\Delta ABE \cong \Delta ACF$ [AAS Criterion]

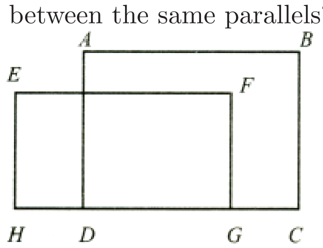
8. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. The largest angle is [1]
(a) 36° (b) 72°
(c) 108° (d) 144°

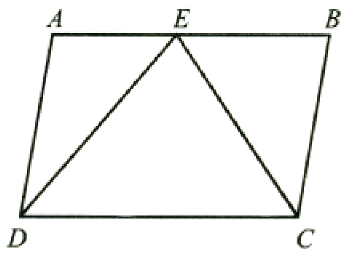
Ans : (d) 144°

Let the angles be x, $2x$, $3x$ and $4x$.

$$x + 2x + 3x + 4x = 360^\circ$$
$$10x = 360^\circ$$
$$x = 36^\circ$$
$$\text{largest angle} = 4x$$
$$= 4 \times 36^\circ$$
$$= 144^\circ$$

9. Which of the following figures lie on the same base and between the same parallels? [1]

(a)


(b)


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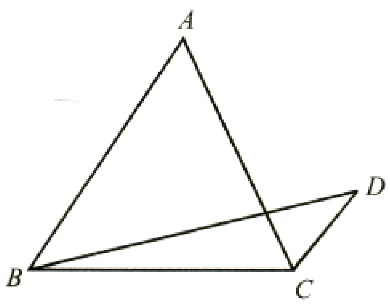
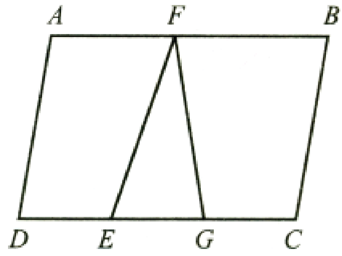


(d)


Ans : (b)

Common base $= DC$ and two parallels are AB and DC.

Thus, $DEDC$ and parallelogram $ABCD$ are on same base DC and between same parallel lines AB and DC.

10. Diagonals of a cyclic quadrilateral are the diameters of that circle, then quadrilateral is a [1]
(a) parallelogram (b) square
(c) rectangle (d) trapezium

Ans : (c) rectangle

(Q.11-Q.15) Fill in the blanks :

11. The construction of a triangle ABC, given that $BC = 6$ cm, $\angle B = 45^\circ$ is not possible when difference of AB and AC is equal to .... cm [1]

Ans : 6.9 cm

It is not possible to construct triangle whose difference of two side is more than the third side.

12. If the perimeter of an equilateral triangle is 90 m, then its area is .... m^2. [1]

Ans : $225\sqrt{3}\ m^2$

Let a be the side of given triangle.

Given, $$3a = 90$$
$$a = 30\text{ m}$$
$$S = \frac{90}{2}$$
$$= 45\text{ m}$$

$$\text{Area of triangle} = \sqrt{45(45-30)(45-30)(45-30)}$$
$$= \sqrt{45 \times 15 \times 15 \times 15}$$
$$= 15 \times 15\sqrt{3}$$
$$= 225\sqrt{3}\ m^2$$

or

If base of a triangle is doubled then its area will be .... times of original area.

Ans : two

13. Volume of a cylinder is three times the volume of a .... on the same base and of the same height. [1]

Ans : cone

14. Width of the class-interval is called .... of class interval. [1]

Ans : size

15. Probability is a measure of .... [1]

Ans : Uncertainty

(Q.16-Q.20) Answer the following :

16. Find a rational number between -5 and -6. [1]

SOLUTION :

$$\text{Rational number} = \frac{-5+(-6)}{2} = \frac{-5-6}{2} = \frac{-11}{2}$$

17. Find the zero of a polynomial $2x+4$ [1]

SOLUTION :

Given polynomial is $p(x) = 2x+4$

On putting $p(x) = 0$

We get $2x+4 = 0$

$$2x = -4$$

$$x = \frac{-4}{2} = -2$$

Hence, $x = -2$

is the zero of the polynomial $2x+4$

18. Find the image of point $(-4,6)$ under origin. [1]

SOLUTION :

If origin is taken as mirror, then signs of both coordinates will be changed.
So, image of point $(-4,6)$ under origin is $(-4,6)$

19. One side of an equilateral triangle is 4 cm Find its area. [1]

SOLUTION :

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4}(a)^2 = 4\sqrt{3}\text{ cm}^2$$

20. Is it correct to say that in a histogram, the area of each rectangle is proportional to the class size of the corresponding class interval? If not, correct the statement. [1]

SOLUTION :

It is not correct, because in a histogram, the area of each rectangle is proportional to the frequency of its class.

Section B

21. Find the value of x, $2^{7x} \div 2^{2x} = \sqrt[5]{2^{15}}$. [2]

SOLUTION :

We have, $2^{7x} \div 2^{2x} = \sqrt[5]{2^{15}}$

$\Rightarrow$ $(2)^{7x-2x} = \sqrt[5]{2^{15}}$

$$2^{5x} = 2^{15 \times \frac{1}{5}}$$

$$= 2^3$$

On comparing the power of 2 from both sides, we get

$\Rightarrow$ $5x = 3$

$$x = \frac{3}{5}$$

or

If $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then find the value of x^2.

SOLUTION :

We have, $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Then, $x^2 = \left(\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)^2$

$$= \frac{3+2+2\sqrt{6}}{3+2-2\sqrt{6}}$$

$$= \frac{5+2\sqrt{6}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

$$= \frac{(5+2\sqrt{6})^2}{25-24}$$

$$= \frac{25+24+20\sqrt{6}}{1}$$

$\Rightarrow$ $x^2 = 49+20\sqrt{6}$

22. Write linear equation such that each point on its graph has ordinate 3 times its abscissa. [2]

SOLUTION :

Let the abscissa of the point be x and the ordinate of the point be y. According to the question,

$$y = 3x$$

when $x = 1$, $y = 3 \times 1 = 3$

when $x = 2$, $y = 3 \times 2 = 6$

when $x = 3$, $y = 3 \times 3 = 9$

Thus, we find three points $A(1,3)$, $B(2,6)$ and $C(3,9)$. Now, we can see that any point on the line joining these points has an ordinate 3 times its abscissa.

23. In which quadrant does the given point lie ? [2]

(i) $A(4,-3)$

(ii) $B(-2,5)$

(iii) $C(-3,-2)$

(iv) $D(2,4)$

SOLUTION :

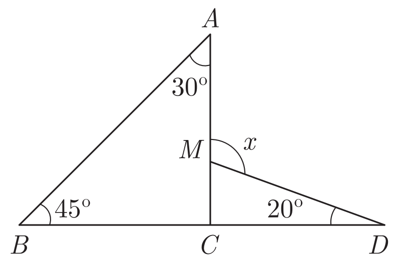
(i) Point $A(4,-3)$ is of the type $(+,-)$.
So, it lies in the IV quadrant.

(ii) Point $B(-2,5)$ is of the type $(-,+)$.
So, it lies in the II quadrant.

(iii) Point $C(-3,-2)$ is of the type $(-,-)$.
So, it lies in the III quadrant.

(iv) Point $D(2,4)$ is of the type $(+,+)$.
So, it lies in the I quadrant.

24. In the given figure, find the value of x. [2]



SOLUTION :

In ΔABC, we have

$$\angle BAC + \angle ABC + \angle ACB = 180°$$

$$\Rightarrow \quad \angle ACB = 105^\circ$$

Also, $\angle ACB + \angle ACD = 180^\circ$

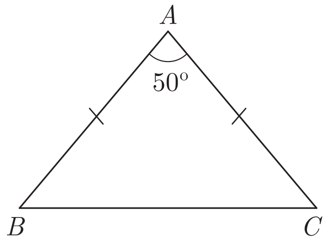
[By linear pair axiom]

$$\Rightarrow \quad \angle ACD = 75^\circ$$

Now, $x = \angle MCD + \angle CDM$

$$x = 75^\circ + 20^\circ = 95^\circ$$

25. In a ΔABC if $AB = 3\text{ cm}$, $AC = 3\text{ cm}$ and $\angle A = 50^\circ$, then find $\angle B$. [2]



SOLUTION :

Given, in ΔABC,

$$AB = AC = 3\text{ cm}$$

$$\therefore \quad \angle C = \angle B$$

But $\angle A + \angle B + \angle C = 180^\circ$

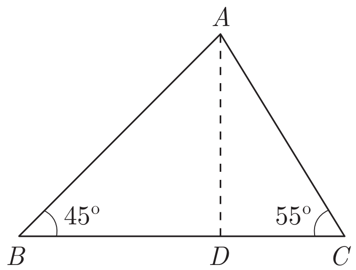
$$\Rightarrow \quad 50^\circ + \angle B + \angle B = 180^\circ$$

$$2\angle B = 130^\circ$$

$$\Rightarrow \quad \angle B = \frac{130^\circ}{2} = 65^\circ$$

or

In a triangle ABC, $\angle B = 45^\circ$, $\angle C = 55^\circ$ and bisector of $\angle A$ meets BC at a point D. Find $\angle ADB$ and $\angle ADC$.



SOLUTION :

$$\angle B = 45^\circ, \quad \angle C = 55^\circ \qquad \text{[Given]}$$

In ΔABC,

$$\Rightarrow \quad \angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$\angle BAC + 45^\circ + 55^\circ = 180^\circ$$

$$\angle BAC = 80^\circ$$

$$\frac{1}{2}\angle BAC = 40^\circ$$

Now, in ΔABD,

$$\angle ADB = 180^\circ - (45^\circ + 40^\circ) = 95^\circ$$

26. A cuboidal water tank is 8 m long, 6 m wide and 3 m deep. How many litres of water can it hold ? [2]

SOLUTION :

Given, the cuboidal water tank have :

length $= 8\text{ m}$,

breadth $= 6\text{ m}$

and height $= 3\text{ m}$

Volume of tank = Capacity of the tank

= Length × Breadth × Height

$= 8\text{ m} \times 6\text{ m} \times 3\text{ m}$

$= 144\text{ m}^3$

Quantity of water which the tank can hold

$= 144 \times 1000\text{ L}$

$[\because \ 1\text{ m}^3 = 1000\text{ L}]$

$= 144000\text{ L}$

or

The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold ? ($1000\text{ cm}^3 = 1\,l$)

SOLUTION :

Given, Height, $h = 25\text{ cm}$,

$$2\pi r = 132\text{ cm}$$

$$2\pi r = 132$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$$

$$r = \frac{132 \times 7}{2 \times 22} = 21\text{ cm}$$

Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 34650\text{ cm}^3$$

$$= \frac{34650}{1000} = 34.65\text{ litres}$$

Section C

27. If $x - y = 5$ and $xy = 84$, find the value of $x^3 - y^3$. [3]

SOLUTION :

$$\begin{aligned} x^3 - y^3 &= (x-y)(x^2 + y^2 + xy) \\ &= (x-y)\left[(x-y)^2 + 2xy + xy\right] \\ &= (x-y)\left[(x-y)^2 + 3xy\right] \\ &= 5\left[(5)^2 + 3 \times 84\right] \\ &= 5[25 + 252] \\ &= 5 \times 277 = 1385 \end{aligned}$$

or

If $2x + 3y = 12$ and $xy = 6$, find the value of $8x^3 + 27y^3$.

SOLUTION :

We know that

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

$$\Rightarrow \quad x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

Now,

$$\begin{aligned} 8x^3 + 27y^3 &= (2x)^3 + (3y)^3 \\ &= (2x+3y)^3 - 3(2x)(3y)(2x+3y) \\ &= 12^3 - 18 \times 6 \times 12 \end{aligned}$$

$$= 1728 - 1296 = 432$$

Hence, $8x^3 + 27y^3 = 432$.

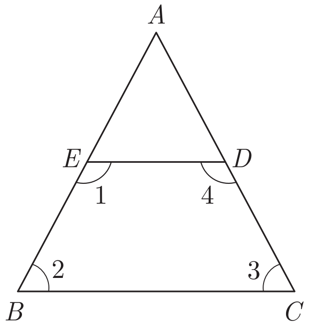
28. If a line is drawn parallel to base of isosceles triangle to intersect its equal sides, then prove that quadrilateral so formed is cyclic. [3]

SOLUTION :

Given : ABC is an isosceles triangle in which $AB = AC$ and $ED || BC$.

To prove : $BCDE$ is a cyclic quadrilateral.

Proof : In ΔABC, $AC = AB$



$\therefore$ $\angle B = \angle C$

or $\angle 2 = \angle 3$

[$\because$ Angles opposite to equal sides of an triangle is equal]

$\because$ $ED || BC$

$\therefore$ $\angle 1 + \angle 2 = 180^\circ$ [Co-interior angles]

$\angle 1 + \angle 3 = 180^\circ$ [$\because$ $\angle 2 = \angle 3$]

$\angle BED + \angle BCD = 180^\circ$ [Co-interior angles] ...(1)

Also, $\angle 3 + \angle 4 = 180^\circ$

$\angle 2 + \angle 4 = 180^\circ$ [$\because$ $\angle 3 = \angle 2$]

$\angle EBC + \angle EDC = 180^\circ$ ...(2)

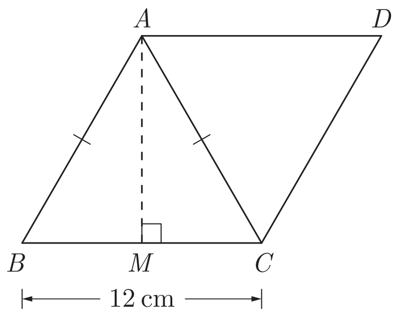
From eqs. (1) and (2), we get quadrilateral $BCDE$ is cyclic.

Hence proved.

29. The perimeter of an isosceles triangle is 32 cm and its base is 12 cm. One of its equal sides forms the diagonal of a parallelogram. Find the area of a parallelogram. [3]

SOLUTION :

Let ΔABC be an isosceles triangle with base $BC = 12\text{ cm}$, perimeter $= 32\text{ cm}$ and AC is a diagonal of a parallelogram $ABCD$. Draw $AM \perp BC$.



So, $BM = CM = \dfrac{12}{2} = 6\text{ cm}$

Also, $AB = AC = \dfrac{32 - 12}{2} = 10\text{ cm}$

and $AM = \sqrt{AC^2 - CM^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{100 - 36}$

$= \sqrt{64} = 8\text{ cm}$

$\therefore$ Area of parallelogram $ABCD$

$= AM \times BC$

$= 8 \times 12 = 96\text{ cm}^2$

or

D and E are the mid-points of BC and AD respectively of ΔABC. If area of $\Delta ABC = 20\text{ cm}^2$, find area of ΔEBD.

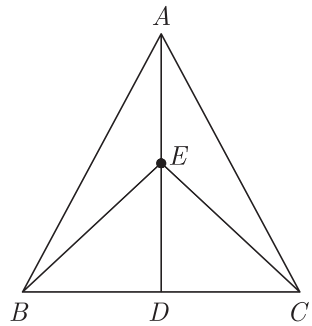
SOLUTION :

Given, D is the mid-point of BC.

$\therefore$ AD is the median of the ΔABC.

$\Rightarrow$ $ar(\Delta ABD) = \dfrac{1}{2} ar(\Delta ABC)$

[$\because$ Median of a triangle divides it into two triangles of equal areas]



$ar(\Delta ABD) = \dfrac{1}{2} \times 20 = 10\text{ cm}^2$

Also, BE is the median of ΔABD

So, $ar(\Delta EBD) = \dfrac{1}{2} ar(\Delta ABD)$

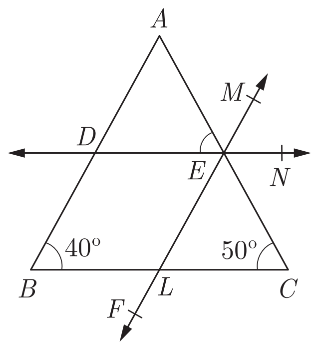
$= \dfrac{1}{2} \times 10 = 5\text{ cm}^2$

30. In the given figure, $DE || BC$ and $MF || AB$. Find : [3]

(i) $\angle ADE + \angle MEN$

(ii) $\angle BDE$

(iii) $\angle BLE$



SOLUTION :

In the given figure, $DE || BC$ and AB is a transversal.

Then $\angle ADE = \angle ABC = 40^\circ$

[$\because$ Corresponding angles are equal]

Also, $AB || MF$ and DE is a transversal.

Then $\angle MEN = \angle ADE$

$\Rightarrow \angle MEN = 40^\circ$

(i) $\angle ADE + \angle MEN = 40^\circ + 40^\circ = 80^\circ$

(ii) $\angle BDE = 180^\circ - \angle ADE$

$= 180^\circ - 40^\circ$

$= 140^\circ$

[$\because$ AB is a line and DE is a ray standing on it, so $\angle ADE + \angle BDE = 180^\circ$]

(iii) $\angle DEL = \angle MEN$

$= 40^\circ$

[Vertically opposite angles]

Now, $DE || BC$ and MF is a transversal.

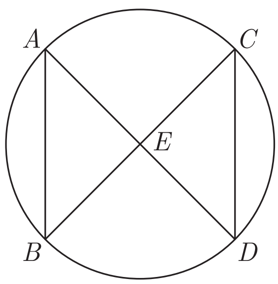
$\therefore \angle DEL + \angle BLE = 180^\circ$

[$\because$ Pair of interior angles on the same side of the transversal are supplementary].

$\Rightarrow 40^\circ + \angle BLE = 180^\circ$

$\angle BLE = 180^\circ - 40^\circ = 140^\circ$

31. In figure, $AB = CD$. Prove that $BE = DE$ and $AE = CE$, where E is the point of intersection of AD and BC. [3]



SOLUTION :

Given : In figure, $AB = CD$. E is the point of intersection of AD and BC.
To prove : $BE = DE$ and $AE = CE$
Proof : In ΔEAB and ΔECD,

$AB = CD$ [Given]

$\angle B = \angle D$ [Angles in the same segment]

$\angle A = \angle C$ [Angles in the same segment]

$\therefore \Delta EAB \cong \Delta ECD$ [By ASA]

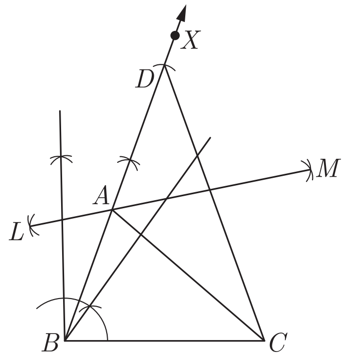
$BE = DE$ [By CPCT]

and $AE = CE$ [By CPCT]

32. Construct a triangle ABC in which $BC = 7\text{ cm}$, $\angle B = 75^\circ$ and $AB + AC = 13\text{ cm}$. [3]

SOLUTION :

Steps of Construction :
(i) Draw a line segment $BC = 7\text{ cm}$.
(ii) At B, draw $\angle CBX = 75^\circ$.
(iii) Cut a line segment $BD = 13\text{ cm}$ from BX.
(iv) Join DC.
(v) Draw the perpendicular bisector LM of CD, which intersects BD at A.
(vi) Join AC. Then ABC is the required triangle.



33. The volume of a cylinder is $448\pi\text{ cm}^3$ and height is 7 cm. Find its lateral surface area and total surface area. [3]

SOLUTION :

Let the radius of the base and height of the cylinder be r cm and h cm respectively.

Then, $h = 7\text{ cm}$ [Given]

Volume $= 448\pi\text{ cm}^3$ [Given]

$\Rightarrow \pi r^2 h = 448\pi$

$\pi \times r^2 \times 7 = 448\pi$

$r^2 = 64$

$\Rightarrow r = 8\text{ cm}$

$\therefore$ Lateral surface area of cylinder

$= 2\pi rh = 2 \times \frac{22}{7} \times 8 \times 7$

$= 352\text{ cm}^2$

Total surface area of cylinder

$= (2\pi rh + 2\pi r^2) = 2\pi r(r+h)$

$= 2 \times \frac{22}{7} \times 8(8+7)$

$= \frac{5280}{7} = 754.28\text{ cm}^2$

or

The largest sphere is carved out of a cube of side 7 cm. Find the volume of the sphere.

SOLUTION :

Side of cube $= 7\text{ cm}$

Diameter of sphere $=$ Side of cube $= 7\text{ cm}$

Radius of sphere $= r = \frac{7}{2}$

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{7}{2}\right)^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= 179.67\text{ cm}^3$

34. Probability of getting a blue ball is $\frac{2}{3}$, from a bag containing 6 blue and 3 red balls. 12 red balls are being added in the bag, then find the probability of getting a blue ball. [3]

SOLUTION :

Given, number of blue balls in a bag $= 6$
Number of red balls in a bag $= 3$

Then, total number of balls in a bag $= 3$
After adding 12 red balls,
Total number of balls became $= 9 + 12 = 21$
Number of blue balls $= 6$

$$P \text{ (getting a blue ball)} = \frac{\text{Number of blue balls}}{\text{Total number of balls}}$$

$$= \frac{6}{21} = \frac{2}{7}$$

Section D

35. If $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$, find the values of a and b. [4]

SOLUTION :

We have,

$$\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$$

$$\Rightarrow \frac{(\sqrt{7}-1)(\sqrt{7}-1)-(\sqrt{7}+1)(\sqrt{7}+1)}{(\sqrt{7}+1)(\sqrt{7}-1)} = a + b\sqrt{7}$$

$$\Rightarrow \frac{(\sqrt{7}-1)^2-(\sqrt{7}+1)^2}{(\sqrt{7})^2-(1)^2} = a + b\sqrt{7}$$

$$\Rightarrow \frac{\{(\sqrt{7})^2-2(\sqrt{7})(1)+(1)^2\}-\{(\sqrt{7})^2+2(\sqrt{7})(1)+(1)^2\}}{7-1}$$

$$= a + b\sqrt{7}$$

$$\Rightarrow \frac{(7-2\sqrt{7}+1)-(7+2\sqrt{7}+1)}{6} = a + b\sqrt{7}$$

$$\Rightarrow \frac{-4\sqrt{7}}{6} = a + b\sqrt{7}$$

$$\Rightarrow -\frac{2}{3}\sqrt{7} = a + b\sqrt{7}$$

$$\Rightarrow a = 0,\ b = -\frac{2}{3}$$

36. Factorise : [4]
$(a+b)^3-(b+c)^3+(c+a)^3+3(a+b)(b+c)(c+a)$

SOLUTION :

$$(a+b)^3-(b+c)^3+(c+a)^3+3(a+b)(b+c)(c+a)$$

$$= (a+b)^3+\{-(b+c)^3+(c+a)^3\}$$
$$-3(a+b)\{-(b+c)\}(c+a)$$

$$= [(a+b)+\{-(b+c)\}+(c+a)]$$
$$[(a+b)^2+\{-(b+c)\}^2+(c+a)^2-(a+b)$$
$$\{-(b+c)\}-\{-(b+c)\}(c+a)-(c+a)(a+b)]$$

$$= (a+b-b-c+c+a)[a^2+2ab+b^2+b^2+2bc$$
$$+c^2+2ca+a^2+(a+b)(b+c)+(b+c)(c+a)$$
$$-(c+a)(a+b)]$$

$$= (2a)[a^2+2ab+b^2+b^2+2bc+c^2$$
$$+c^2+2ca+a^2+ab+b^2+bc+ac+bc$$
$$+ba+c^2+ca-ca-cb-a^2-ab]$$

$$= 2a[a^2+3b^2+3c^2+3ab+3bc+3ca]$$

or

If $a+b+c=0$, then prove that

$$\frac{(b+c)^2}{3bc}+\frac{(c+a)^2}{3ac}+\frac{(a+b)^2}{3ab} = 1$$

SOLUTION :

$$\text{L.H.S.} = \frac{(b+c)^2}{3bc}+\frac{(c+a)^2}{3ac}+\frac{(a+b)^2}{3ab}$$

$$= \frac{b^2+c^2+2bc}{3bc}+\frac{c^2+a^2+2ac}{3ac}+\frac{a^2+b^2+2ab}{3ab}$$

[Using $(x+y)^2 = x^2+y^2+2xy$]

$$= \frac{1}{3abc}[ab^2+ac^2+2abc+bc^2+ba^2+2abc+a^2c$$
$$+b^2c+2abc]$$

$$= \frac{1}{3abc}[ab^2+ac^2+bc^2+ba^2+a^2c+b^2c+6abc]$$

$$= \frac{1}{3abc}[ab(b+a)+ac(c+a)+bc(c+b)+6abc]$$

$$= \frac{1}{3abc}[ab(-c)+ac(-b)+bc(-a)+6abc]$$

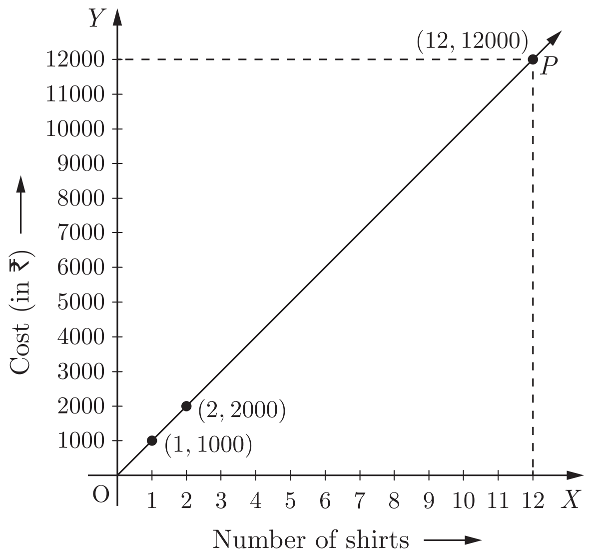
$[\because\ a+b+c=0]$

$$= \frac{1}{3abc}[-abc-abc-abc+6abc]$$

$$= \frac{3abc}{3abc} = 1 = \text{R.H.S.}$$ Hence proved

37. The cost of a shirt of a particular brand is ₹ 1000. Write a linear equation, when the cost of x shirts is ₹ y. Draw the graph of this equation and find the cost of 12 such shirts from the graph. [4]

SOLUTION :



Given, cost of x shirts $=$ ₹ y

$\therefore$ Cost of 1 shirt $=$ ₹ $\frac{y}{x}$

According to the question,

$$\Rightarrow \frac{y}{x} = 1000$$

$$1000x - y = 0$$

From graph, the cost of 12 shirts is ₹ 12000.

38. Construct a triangle ABC in which $BC = 5.8\,\text{cm}$, $\angle B = 45^\circ$ and $\angle C = 60^\circ$. Construct angle bisectors of $\angle B$ and $\angle C$ and intersect them at point O. Measure $\angle BOC$. [4]

SOLUTION :

Steps of construction :

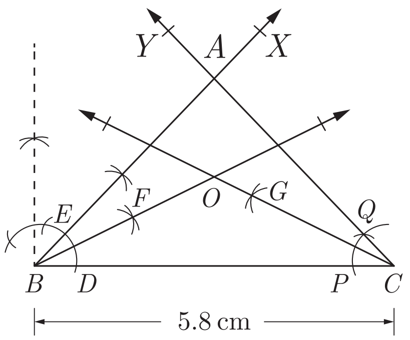
(a) Draw a line segment $BC = 5.8\,\text{cm}$.

(b) At B and C, draw $\angle XBC = 45^\circ$ and $\angle YCB = 60^\circ$.

(c) The rays XB and YC intersect at A. Therefore, ΔABC is the required triangle.

(d) Taking B as centre, and with some radius, draw arcs intersecting XB and BC at E and D respectively.

(e) Taking D and E as centres with radius greater than $\frac{1}{2}DE$, draw arcs intersecting each other at F.



(f) Draw the ray BF. It is the angle bisector of $\angle B$.

(g) Similarly, construct angle bisector CG of $\angle C$.

(h) Let BF and CG intersect each other at O.

(i) On measuring $\angle BOC$, we get, $\angle BOC = 127.5^\circ$.

39. The outer diameter of a spherical shell is 10 cm and the inner diameter is 9 cm. Find the volume of the metal contained in the shell. (Use $\pi = \frac{22}{7}$) [4]

SOLUTION :

$$\text{Outer diameter} = 10\,\text{cm}$$

$$\therefore \text{ Outer radius } (R) = \frac{10}{2} = 5\,\text{cm}$$

$$\text{Inner diameter} = 9\,\text{cm}$$

$$\text{Inner radius } (r) = \frac{9}{2}\,\text{cm}$$

Volume of the metal contained in the shell

$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7}\left[(5)^3 - \left(\frac{9}{2}\right)^3\right]$$

$$= \frac{4}{3} \times \frac{22}{7} \times \left[125 - \frac{729}{8}\right]$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{271}{8}$$

$$= \frac{2981}{21}\,\text{cm}^3$$

40. The runs scored by two teams A and B on the first 60 balls in a cricket match are given below : [4]

Number of balls	Team A	Team B
1 - 6	2	5
7 - 12	1	6
13 - 18	8	2
19 - 24	9	10
25 - 30	4	5
31 - 36	5	6
37 - 42	6	3
43 - 48	10	4
49 - 54	6	8
55 - 60	2	10

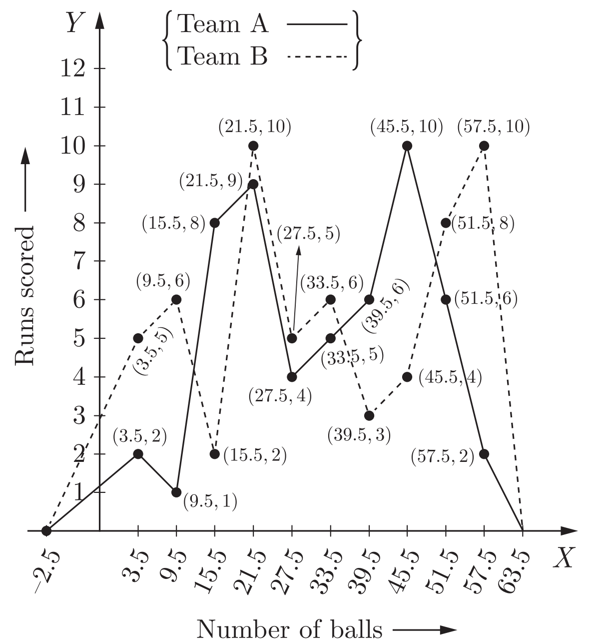
Represent the data of both the teams on the same graph by frequency polygons.

SOLUTION :

First, we make the class intervals continuous then modified table of given data is as shown below.

Number of balls	Class marks	Team A	Team B
0.5 - 6.5	3.5	2	5
6.5 - 12.5	9.5	1	6
12.5 -18.5	15.5	8	2
18.5 - 24.5	21.5	9	10
24.5 - 30.5	27.5	4	5
30.5 - 36.5	33.5	5	6
36.5 - 42.5	39.5	6	3
42.5 - 48.5	45.5	10	4
48.5 - 54.5	51.5	6	8
54.5 - 60.5	57.5	2	10

Now, frequency polygon for both teams are given below



or

Draw a histogram and frequency polygon on the same graph for the following data.

Class interval	**Frequency**
150 - 200	5
200 - 250	3
250 - 300	5
300 - 350	6
350 - 400	8
400 - 450	7
450 - 500	1

SOLUTION :