CHAPTER 13

Surface Areas and Volumes

1. OBJECTIVE QUESTIONS

1. Vertical cross-section of a right circular cylinder is always a
(a) rectangle (b) rhombus
(c) square (d) trapezium

Ans : (a) rectangle

2. The ratio of the volume and surface area of a sphere of unit radius without considering units is
(a) 4 : 3 (b) 3 : 4
(c) 1 : 3 (d) 3 : 1

Ans : (c) 1 : 3

$$r = 1 \text{ unit (given)}$$

$$\frac{\text{Volume of sphere}}{\text{Surface area of sphere}} = \frac{\frac{4}{3}\pi(1)^3}{4\pi(1)^2}$$

$$= \frac{1}{3} = 1:3$$

3. The number of surfaces in right cylinder is
(a) 4 (b) 3
(c) 2 (d) 1

Ans : (b) 3

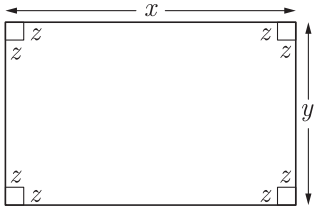
No. of surfaces in cylinder $= 3$

4. A rectangular sheet of metal, x cm by y cm has a square of size z cm cut from each corner. The sheet is then bent to form a tray of depth z cm. The volume of the tray is
(a) $z(x-z)(y-z)\,\text{cm}^3$ (b) $xyz\,\text{cm}^3$
(c) $z(x-2z)(y-2z)\,\text{cm}^3$ (d) $(x+y)z\,\text{cm}^3$

Ans : (c) $z(x-2z)(y-2z)\,\text{cm}^3$

When rectangular sheet is cut at each corner, then length, breadth and height respectively will be $x-2z, y-2z, z$

Hence, Volume of tray $= (x-2z)(y-2z)z$



5. A metallic right circular cone of height 9 cm and base radius 7 cm is melted and recast into a cuboid whose two sides are 11 cm and 6 cm. Then the third side of the cuboid is
(a) 5 cm (b) 6 cm
(c) 7 cm (d) 10 cm

Ans : (c) 7 cm

Let h be the third side of the cuboid.
Volume of right circular cone=Volume of cuboid

$$\frac{1}{3}\pi(7)^2 \times 9 = 11 \times 6 \times h$$

$$h = \frac{1 \times 22 \times 7 \times 7 \times 9}{3 \times 7 \times 11 \times 6}$$

$$h = 7 \text{ cm}$$

6. Two cylindrical jars have their diameters in the ratio 3 : 1, but height 1 : 3. Then the ratio of their volumes is
(a) 1 : 3 (b) 3 : 1
(c) 2 : 5 (d) 1 : 4

Ans : (b) 3 : 1

$$\text{Required ratio} = \frac{\pi\left(\frac{3}{2}\right)^2 \cdot 1}{\pi\left(\frac{1}{2}\right)^2 \cdot 3}$$

$$= \frac{9}{4} \times \frac{4}{3} = \frac{3}{1} \equiv 3:1$$

7. If the radius and height of a right circular cone are 'r' and 'h' respectively, the slant height of the cone is
(a) $(h^2+r^2)^{1/3}$ (b) $(h+r)^{1/3}$
(c) $(h^2+r^2)^{1/2}$ (d) none of these

Ans : (c) $(h^2+r^2)^{1/2}$

$$\ell = \sqrt{h^2+r^2}$$

8. Two circular cylinders of equal volume have their heights in the ratio 1 : 2 Ratio of their radii is
(a) $\sqrt{2}:1$ (b) 1 : 2
(c) 1 : 4 (d) $1:\sqrt{2}$

Ans : (a) $\sqrt{2}:1$

Given, $\pi r_1^2 h_1 = \pi r_2^2 h_2$

$$\pi r_1^2(1) = \pi r_2^2(2)$$

$$\frac{r_1^2}{r_2^2} = \frac{2}{1}$$

$$\frac{r_1}{r_2} = \frac{\sqrt{2}}{1} \equiv \sqrt{2}:1$$

9. If V is the volume of a cuboid of dimensions a, b, c and A is its surface area, then $\frac{A}{V}$ is
(a) $a^2b^2c^2$ (b) $2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$
(c) $\frac{1}{abc}$ (d) $\frac{1}{2}\left(\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}\right)$

Ans : (b) $2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$$A = 2(ab+bc+ca)$$
$$V = abc$$
Hence, $$\frac{A}{V} = \frac{2(ab+bc+ca)}{abc}$$
$$= 2\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)$$

10. The volumes of two cylinders are as $a:b$ and their heights are as $c:d$. Find the ratio of their diameters.

(a) $\frac{ad}{bc}$ (b) $\frac{ad^2}{bc^2}$

(c) $\sqrt{\frac{ad}{bc}}$ (d) $\sqrt{\frac{a}{b}\times\frac{c}{d}}$

Ans : (c) $\sqrt{\frac{ad}{bc}}$

Let r_1, r_2 be the radii of the two cylinders respectively. Let the heights of the two cylinders be cK and dK respectively.

Hence, $$\frac{\pi r_1^2 cK}{\pi r_1^2 dK} = \frac{a}{b}$$
$$\frac{r_1}{r_2} = \sqrt{\frac{ad}{bc}}$$

11. How many metres of cloth 5 m wide, will be required to make a conical tent, if the radius of whose base is 7 m and height is 24 m?

(a) 120 m (b) 110 m

(c) 100 m (d) 105 m

Ans : (b) 110 m

Let l be the length of the cloth required to make a conical tent.

According to question,

Area of cloth = Curved surface area of conical tent

$$l\times 5 = \pi\times 7\times\sqrt{(7)^2+(24)^2}$$
$$l = \frac{22\times 7\times 25}{7\times 5}$$
$$l = 100\,\text{m}$$

12. If the height of a cone is doubled, then its volume is increased by

(a) 100% (b) 200%

(c) 300% (d) 400%

Ans : (a) 100%

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$\text{New volume} = \frac{1}{3}\pi r^2(2h) = \frac{2}{3}\pi r^2 h$$
[height of cone $= 2h$]

$$\%\text{ increase in volume} = \frac{\left(\frac{2}{3}-\frac{1}{3}\right)\pi r^2 h}{\frac{1}{3}\pi r^2 h}\times 100 = 100\%.$$

13. The length of longest pole that can be placed on the floor of room is 10 m and the length of the longest pole that can be placed in the room is $10\sqrt{2}$ m. The height of the room is

(a) 6 m (b) 7.5 m

(c) 8 m (d) 10 m

Ans : (d) 10 m

We have, $$l^2+b^2 = (10)^2 = 100$$
and $$l^2+b^2+h^2 = \left(10\sqrt{2}\right)^2 = 200$$
$$h^2 = (200-100) = 100$$
$$h = 10\,\text{m}$$

14. If each edge of a cube, of volume V, is doubled, then the volume of the new cube is

(a) 2 V (b) 4 V

(c) 6 V (d) 8 V

Ans : (d) 8 V

Let 'a' be the edge of cube.

$$(V)\text{ volume of cube} = a^3$$
$$\text{Edge of new cube} = 2a$$
$$\text{Volume of cube} = (2a)^3 = 8a^3 = 8\,(\text{V})$$

15. If the perimeter of one face of a cube is 20 cm, then its surface area is

(a) $120\,\text{cm}^2$ (b) $150\,\text{cm}^2$

(c) $125\,\text{cm}^2$ (d) $400\,\text{cm}^2$

Ans : (b) $150\,\text{cm}^2$

$$\text{Edge of cube} = \frac{20}{4}\,\text{cm} = 5\,\text{cm}$$
$$\text{Surface area} = 6\times 5^2\,\text{cm}^2 = 150\,\text{cm}^2$$

16. The dimensions of an open box are 52 cm, 40 cm and 29 cm. Its thickness is 2 cm. If $1\,\text{cm}^3$ of metal used in the box weighs 0.5 gms, the weight of the box is

(a) 8.56 kg (b) 7.76 kg

(c) 7.576 kg (d) 6.832 kg

Ans : (d) 6.832 kg

Volume of metal
$$= (52\times 40\times 29 - 48\times 36\times 27)\,\text{cm}^3$$
$$= 13664\,\text{cm}^3$$
Weight of the box
$$= \left(\frac{13664\times 0.5}{1000}\right)\text{kg} = 6.832\,\text{kg}$$

17. The area of the cardboard needed to make a box of size $25\,\text{cm}\times 15\,\text{cm}\times 8\,\text{cm}$ will be

(a) $390\,\text{cm}^2$ (b) $1000\,\text{cm}^2$

(c) $1390\,\text{cm}^2$ (d) $2780\,\text{cm}^2$

Ans : (c) $1390\,\text{cm}^2$

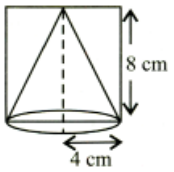
Area of cardboard needed
$$= 2(25\times 15+15\times 8+25\times 8)\,\text{cm}^2$$
$$= 1390\,\text{cm}^2$$

18. The volume of the largest circular cone that can be cut from a cube whose edge is 8 cm, is

(a) $131.09\,\text{cm}^3$ (b) $132.09\,\text{cm}^3$

(c) $133.09\,\text{cm}^3$ (d) $134.09\,\text{cm}^3$

Ans : (d) $134.09\,\text{cm}^3$



$$h = 8\,\text{cm},\ r = 4\,\text{cm}$$

$$\text{Volume of largest cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (4)^2 \times 8$$

$$= \frac{2816}{21} = 134.09\,\text{cm}^3$$

19. The percentage increase in the surface area of a cube when each side is doubled, is

(a) 25% (b) 50%

(c) 150% (d) 300%

Ans : (d) 300%

Surface area of a cube having side 'a' is $6a^2$. If each side is doubled, then the new surface area $= 6(2a)^2$

$$= 6 \times 4a^2$$

$$\%\text{increase in surface area} = \frac{\text{change in area}}{\text{Original area}} \times 100$$

$$= \frac{6a^2(4-1)}{6a^2} \times 100$$

$$= 300\%$$

20. A cuboidal wooden box has the inner dimensions measures as 115 cm, 75 cm, 35 cm and the thickness of wood is 2.5 cm. Then the volume of the wood is

(a) 80,000 cm^3 (b) 82,125 cm^3

(c) 84,000 cm^3 (d) 85,000 cm^3

Ans : (b) 82,125 cm^3

Internal dimensions of the box are 115 cm, 75 cm and 35 cm.

$$\text{Internal volume of the box} = l \times b \times h$$

$$= 115 \times 75 \times 35$$

$$= 301875\,\text{cm}^3$$

Since, thickness of the wood $= 2.5\,\text{cm}$.

Hence, external dimensions are $(115+5)\,\text{cm}, (75+5)\,\text{cm}$ and $(35+5)\,\text{cm}$.

i.e., 120 cm, 80 cm, 40 cm

$$\text{External volume of the box} = l \times b \times h$$

$$= 120 \times 80 \times 40$$

$$= 384000\,\text{cm}^3$$

Volume of the wood,

$$= \text{External volume} - \text{Internal volume}$$

$$= 384000 - 301875 = 82125\,\text{cm}^3$$

21. Volume of a hemisphere is 19404 cubic cm. The total surface area is

(a) 2772 sq. cm (b) 4158 sq. cm

(c) 5544 sq. cm (d) 1386 sq. cm

Ans : (b) 4158 sq. cm

$$\text{Volume of the hemisphere} = 19404\,\text{cm}^3$$

$$\frac{2}{3}\pi r^3 = 19404$$

$$r^3 = \frac{3 \times 19404 \times 7}{22 \times 2}$$

$$r^3 = (21)^3$$

$$r = 21\,\text{cm}$$

Total surface area of hemisphere

$$= 3\pi r^2$$

$$= 3 \times \frac{22}{7} \times 21 \times 21$$

$$= 9 \times 22 \times 21$$

$$= 189 \times 22$$

$$= 4158\,\text{sq. cm}$$

22. The edge of a cube is 20 cm. How many small cubes of edge 5 cm can be formed from this cube?

(a) 4 (b) 32

(c) 64 (d) 100

Ans : (c) 64

Let 'n' be the number of cubes which can be formed from the given cube.

$$\text{Volume of big cube} = \text{Volume of } n \text{ smaller cubes}$$

$$20 \times 20 \times 20 = n \times 5 \times 5 \times 5$$

$$n = \frac{20 \times 20 \times 20}{5 \times 5 \times 5}$$

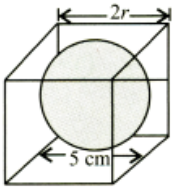
$$= 4 \times 4 \times 4 = 64$$

23. The largest sphere is cut off from a cube of side 5 cm. The volume of the sphere will be

(a) $27\pi\,\text{cm}^3$ (b) $30\pi\,\text{cm}^3$

(c) $108\pi\,\text{cm}^3$ (d) $\frac{125\pi}{6}\,\text{cm}^3$

Ans : (d) $\frac{125\pi}{6}\,\text{cm}^3$



Let r be the radius of the largest sphere that can be cut off from the cube.

Then, diameter of sphere $= 5\,\text{cm}$

$$2r = 5$$

$$r = \frac{5}{2}\,\text{cm}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times \frac{5 \times 5 \times 5}{2 \times 2 \times 2}$$

$$= \frac{125\pi}{6}\,\text{cm}^3$$

24. If the volumes of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5, then the ratio of their heights is

(a) 1 : 5 (b) 5 : 4

(c) 5 : 16 (d) 25 : 64

Ans : (d) 25 : 64

Let diameters of two cones be $4k$ and $5k$ respectively.
Then radii of two cones are $2k$ and $\frac{5k}{2}$ respectively.
Let h_1, h_2 be the heights of two cones respectively.

Ratio of volume of two cones $= 1:4$

or
$$\frac{\frac{1}{3}\pi(2k)^2 h_1}{\frac{1}{3}\pi\left(\frac{5}{2}k\right)^2 h_2} = \frac{1}{4}$$
$$\frac{4\times 4}{25}\frac{h_1}{h_2} = \frac{1}{4}$$
$$\frac{h_1}{h_2} = \frac{25}{4\times 16}$$
$$= \frac{25}{64} = 25:64$$

25. A cylindrical vessel 60 cm in diameter is partially filled with water. A sphere of 60 cm in diameter is gently dropped into the vessel. To what further height will water rise in the cylinder?
(a) 15 cm (b) 30 cm
(c) 40 cm (d) 25 cm

Ans : (c) 40 cm

Let h be the height of displaced water. Radius of both cylindrical vessel and sphere is $\frac{60}{2} = 30$ cm
According to question,

Volume of displaced water = Volume of sphere
$$\pi(30)^2 h = \frac{4}{3}\pi(30)^2$$
$$h = \frac{4\times 30}{3} = 40\text{ cm}$$

26. If the length of diagonal of a cube is $\sqrt{12}$ cm, then the volume of the cube is
(a) $8\sqrt{12}\,\text{cm}^3$ (b) $8\,\text{cm}^3$
(c) $16\sqrt{2}\,\text{cm}^3$ (d) $16\,\text{cm}^3$

Ans : (b) $8\,\text{cm}^3$

Let 'a' be the the side of cube.

Since length of diagonal $= \sqrt{12}$ cm
Hence, $\sqrt{a^2+a^2+a^2} = \sqrt{12}$
Squaring both sides, we get
$$3a^2 = 12$$
$$a^2 = 4$$
$$a = 2$$
Hence, Volume of cube $= 2\times 2\times 2 = 8\,\text{cm}^3$

27. The surface area of a cube is $726\,\text{m}^2$. Its volume is
(a) $1300\,\text{m}^3$ (b) $1331\,\text{m}^3$
(c) $1452\,\text{m}^3$ (d) $1542\,\text{m}^3$

Ans : (b) $1331\,\text{m}^3$

Let 'a' be the side of the cube.
$$\text{Surface area} = 6a^2 = 726$$
$$a^2 = 121$$
$$a = 11\,\text{m}$$
$$\text{Volume of the cube} = (11\times 11\times 11)\,\text{m}^3 = 1331\,\text{m}^3$$

28. The length of longest rod that can be placed in a room 20 m long, 16 m broad and 12 m high, is
(a) 20 m (b) 16.4 m
(c) 48 m (d) 28.2 m

Ans : (d) 28.2 m

$$\text{Length of the longest rod} = \sqrt{l^2\times b^2\times h^2}$$
$$= \sqrt{(20)^2+(16)^2+(12)^2}$$
$$= \sqrt{800}$$
$$= 20\sqrt{2}$$
$$= (20\times 1.41) = 28.2\,\text{m}$$

29. The length of diagonal of cube is $(14\times\sqrt{3})$ cm. The volume of the cube is
(a) $2744\sqrt{3}\,\text{cm}^3$ (b) $2744\,\text{cm}^3$
(c) $588\,\text{cm}^3$ (d) $3528\,\text{cm}^3$

Ans : (b) $2744\,\text{cm}^3$

Let 'a' be the side of cube.
$$\text{Length of diagonal} = \sqrt{3}\,a = 14\times\sqrt{3}$$
$$a = 14\,\text{cm}$$
$$\text{Volume of cube} = (14\times 14\times 14)\,\text{cm}^3$$
$$= 2744\,\text{cm}^3$$

30. If the length, breadth and height of a cuboid are in the ratio $6:5:4$ and if the total surface area is $33300\,\text{cm}^2$, then the length, breadth and height (in cms) respectively are
(a) 90, 85, 60 (b) 85, 75, 60
(c) 90, 75, 70 (d) 90, 75, 60

Ans : (d) 90, 75, 60

Let, length $= 6x$, breadth $= 5x$
and height $= 4x$
$$\text{T.S.A.} = 2(6x\times 5x + 5x\times 4x + 6x\times 4x)$$
$$= 148x^2$$
$$148x^2 = 33300$$
$$x^2 = \frac{33300}{148} = 225$$
$$x = 15$$
$$\text{Length} = 90\,\text{cm},\ \text{Breadth} = 75\,\text{cm},$$
$$\text{Height} = 60\,\text{cm}.$$

31. A metal sheet 27 cm long, 8 cm broad and 1 cm thick is melted and recast into a cube. The difference between the surface areas of two solids is
(a) $284\,\text{cm}^2$ (b) $286\,\text{cm}^2$
(c) $296\,\text{cm}^2$ (d) $300\,\text{cm}^2$

Ans : (b) $286\,\text{cm}^2$

$$\text{Volume of sheet} = (27\times 8\times 1)\,\text{cm}^3$$
$$= 216\,\text{cm}^3$$
$$\text{Volume of cube formed} = 216\,\text{cm}^3$$
$$\text{Edge of the cube} = (6\times 6\times 6)^{1/3} = 6\,\text{cm}$$
Surface area of metal sheet
$$= 2(27\times 8 + 8\times 1 + 27\times 1)\,\text{cm}^2$$

$= 502\,\text{cm}^2$

Surface area of the cube formed

$= \left[6 \times (6)^2\right]\text{cm}^2 = 216\,\text{cm}^2$

Difference in areas $= (502 - 216)\,\text{cm}^2 = 286\,\text{cm}^2$

32. Bricks are worth ₹750 per 1000 pieces and their length, breadth and height are 25 cm, 12.5 cm, and 7.5 cm respectively. The cost of bricks required to build a wall 200 m long, 1.8 m high and 37.5 cm thick, is

(a) ₹42600 (b) ₹43200

(c) ₹40750 (d) ₹41860

Ans : (b) ₹43200

Number of bricks

$$= \frac{\text{Volume of wall in cm}^3}{\text{Volume of 1 brick}}$$

$$= \left(\frac{200 \times 100 \times 1.8 \times 100 \times 37.5}{25 \times 12.5 \times 7.5}\right)$$

$$= 57600$$

Cost of bricks to build the wall

$$= \left(\frac{750 \times 57600}{1000}\right) = ₹\ 43200$$

33. A tank 3 m long, 2 m wide and 1.5 m deep is dug in a field 22 m long and 14 m wide. If the earth dugout is evenly spread out over the field, the rise in level of the field will be

(a) 0.299 cm (b) 0.29 cm

(c) 2.98 cm (d) 4.15 cm

Ans : (c) 2.98 cm

Earth dug out $= (3 \times 2 \times 1.5)\,\text{m}^3 = 9\,\text{m}^3$

Area on which earth has been spread

$$= (22 \times 14 - 3 \times 2)\,\text{m}^2$$

$$= 302\,\text{m}^2$$

$$\text{Rise in level of field} = \frac{\text{Volume}}{\text{Area}}$$

$$= \left(\frac{9}{302}\right)\text{m} = \left(\frac{9 \times 100}{302}\right)\text{cm}$$

$$= 2.98\,\text{cm}$$

34. The radius of a circular cylinder is same as that of a sphere. Their volumes are equal. The height of the cylinder is

(a) $\frac{4}{3}$ times its radius (b) $\frac{2}{3}$ times its radius

(c) equal to its radius (d) equal to its diameter

Ans : (a) $\frac{4}{3}$ times its radius

Let h be the height of cylinder.

Radius of cylinder = Radius of sphere $= r$

Then, we have $\frac{4}{3}\pi r^3 = \pi r^2 h$

$$h = \frac{4}{3}r$$

Height of cylinder $= \frac{4}{3}$ times its radius.

35. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by

(a) 4.5 cm (b) 2.25 cm

(c) $\frac{4}{9}$ cm (d) $\frac{2}{9}$ cm

Ans : (b) 2.25 cm

$$\pi \times (4)^2 \times h = \frac{4}{3}\pi \times (3)^3$$

$$h = \frac{9}{4}\,\text{cm} = 2.25\,\text{cm}$$

36. If the inner dimensions of a cuboidal box are $50\,\text{cm} \times 40\,\text{cm} \times 30\,\text{cm}$, then the length of the longest rod that can be placed in the box is

(a) 50 cm (b) $50\sqrt{2}$ cm

(c) $50\sqrt{3}$ cm (d) 120 cm

Ans : (b) $50\sqrt{2}$ cm

Length of longest rod = length of diagonal

$$= \sqrt{\ell^2 + b^2 + h^2}$$

$$= \sqrt{(50)^2 + (40)^2 + (30)^2}$$

$$= 50\sqrt{2}\ \text{cm}$$

37. A cylindrical pencil of diameter 1.2 cm has one of its end sharpened into a conical shape of height 1.4 cm. The volume of the material removed is

(a) $1.056\,\text{cm}^3$ (b) $4.224\,\text{cm}^3$

(c) $10.56\,\text{cm}^3$ (d) $42.24\,\text{cm}^3$

Ans : (a) $1.056\,\text{cm}^3$

Volume of material removed

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \pi r^2 h\left[\frac{2}{3}\right]$$

$$= \frac{22}{7} \times (0.6)^2 \times 1.4 \times \frac{2}{3}$$

$$= 1.056\,\text{cm}^3$$

38. A small indoor greenhouse is made entirely of glass panes held together with tape. It is 30 cm long, 25 cm wide and 25 cm high. How much of tape is needed for all the 12 edges?

(a) 230 cm (b) 320 cm

(c) 302 cm (d) 203 cm

Ans : (b) 320 cm

Length of tape

$$= (4l + 4b + 4h)\,\text{cm}$$

$$= (4 \times 30 + 4 \times 25 + 4 \times 25)\,\text{cm}$$

$$= 320\,\text{cm}$$

39. A conical tent has a floor area of 154 sq. m. Its height is 24 m. How much canvas is required for the tent?

(a) 500 sq. m. (b) 550 sq. m.

(c) 700 sq. m. (d) 450 sq. m.

Ans : (b) 550 sq. m.

Area of base $= 154$

$$\pi r^2 = 154$$

$$\frac{22}{7} \times r^2 = 154$$

$$r^2 = \frac{154 \times 7}{22}$$

$$r = 7\,\text{m}$$

$$\text{Slant height of cone} = \sqrt{r^2+h^2} = \sqrt{7^2+24^2}$$

$$= \sqrt{625} = 25\,\text{m}$$

$$\text{Curved surface area} = \pi rl$$

$$= \frac{22}{7} \times 7 \times 25$$

$$= 550\,\text{m}^2$$

40. A hemispherical tank of radius 3 cm is full of milk. It is connected to a pipe, through which liquid is emptied at the 1/7 litre per second. The time taken to empty the tank completely?

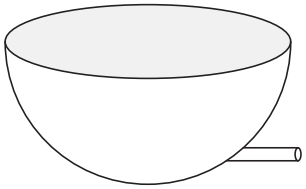
(a) 0.302 sec (b) 0.396 sec

(c) 0.453 sec (d) 0.492 sec

Ans : (b) 0.396 sec

Radius of the hemisphere

$$h = 3\,\text{cm}$$



Volume of milk in the sphere

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 3^3$$

$$= \frac{22}{7} \times 2 \times 3^2\,\text{cm}^3$$

$$= \frac{22}{7} \times 2 \times 3^2 \times \frac{1}{1000} \cdot \text{litre}$$

Flow of liquid through the pipe $= \frac{1}{7}$ litre per second.

Time taken to empty the hemisphere

$$= \frac{22}{7} \times 2 \times 3^2 \times \frac{1}{1000} \times 7$$

$$= \frac{396}{1000}\,\text{sec}.$$

41. The radius of a sphere is increased by $P\%$. Its surface area increases by

(a) $P\%$ (b) $P^2\%$

(c) $\left(2P+\frac{P^2}{100}\right)\%$ (d) $\frac{P^2}{2}\%$

Ans : (c) $\left(2P+\frac{P^2}{100}\right)\%$

Let r be radius of the sphere

$$\text{New radius} = \left(r+\frac{P}{100}r\right) = \left(1+\frac{P}{100}\right)r$$

Original surface area of sphere $= 4\pi r^2$

$$\text{New surface area} = 4\pi\left(\left(1+\frac{P}{100}\right)r\right)^2$$

$$= 4\pi\left(1+\frac{P}{100}\right)^2 r^2$$

Hence, % increase in surface area

$$= \frac{\text{Change in surface area}}{\text{Original surface area}} \times 100$$

$$= \left(\left(1+\frac{P}{100}\right)^2 - 1\right) \times 100$$

$$= \left(1+\frac{P^2}{(100)^2}+\frac{2P}{100}-1\right) \times 100$$

$$= \left(\frac{P^2}{100}+2P\right)\%$$

42. If the volumes of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5, then the ratio of their heights

(a) 25 : 64 (b) 5 : 4

(c) 64 : 25 (d) 5 : 16

Ans : (a) 25 : 64

$$\frac{\frac{1}{3}\pi(2)^2 h_1}{\frac{1}{3}\pi\left(\frac{5}{2}\right)^2 h_2} = \frac{1}{4}$$

$$\frac{4h_1}{\frac{25}{4}h_2} = \frac{1}{4}$$

$$\frac{16h_1}{25h_2} = \frac{1}{4}$$

$$\frac{h_1}{h_2} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}$$

2. FILL IN THE BLANK

DIRECTION : Complete the following statements with an appropriate word/term to be filled in the blank space(s).

1. The sum of the areas of the plane and curved surfaces (faces) of a solid is called its ..... surface area.

Ans : total

2. A sphere has only ..... surface and that is curved.

Ans : One

3. The curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm is .....

Ans : $220\,\text{cm}^2$

4. Volume of a cylinder is three times the volume of a ..... on the same base and of the same height.

Ans : cone

5. ..... of a solid is the amount of space enclosed by the bounding surface.

Ans : Volume

6. The solid bounded by two concentric spherical surfaces is called a .....

Ans : Spherical shell

7. The volume of a sphere is ..... to two-thirds the volume of a cylinder of the same height and diameter.

Ans : equal

8. A right circular cone is generated by revolving a right angled triangle about one of the sides containing the ..... .

Ans : right angle

9. The volume of a rectangular solid measuring 1 m by 50 cm by 0.5 m is ..... cm^3.

Ans : 250, 000

10. Cube is a special form of ..... .

Ans : Cuboid

3. TRUE/FALSE

DIRECTION : Read the following statements and write your answer as true or false.

1. In a sphere the number of faces is 2.

Ans : False, sphere has 1 face.

2. If the line joining the centres of circular ends of a cylinder is not perpendicular to the circular ends, then the cylinder is not a right circular cylinder.

Ans : True

3. The curved surface area of a cone is also called the lateral surface area.

Ans : True

4. A right circular cone is a solid generated by revolving a line segment which passes through a fixed point and which makes a constant angle with a fixed line.

Ans : True

5. A solid generated by the revolution of a rectangle about one of its sides is called a right circular cylinder.

Ans : True

6. Each of the circular ends on which the cylinder rests is called its base.

Ans : True

4. MATCHING QUESTIONS

DIRECTION : In the section, each question has two matching lists. Choices for the correct combination of elements from Column-I and Column-II are given as options (a), (b), (c) and (d) out of which one is correct.

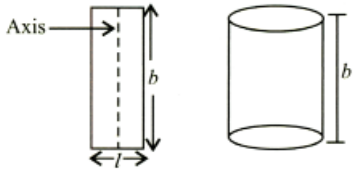
1. Match the description given in Column-I with their corresponding solid figure in Column-II.

Column-I		Column-II	
(P)	Rectangle rotating about axis	(1)	cone
(Q)	Semicircle rotating about diameter	(2)	cube
(R)	Right angled triangle rotating about base	(3)	cylinder
(S)	Six square sheet	(4)	sphere

	P	Q	R	S
(a)	1	2	3	4
(b)	3	4	1	2
(c)	4	2	3	1
(d)	3	1	2	4

Ans : (b) P − 3, Q − 4, R − 1, S − 2

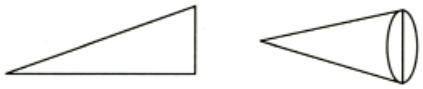
(P)



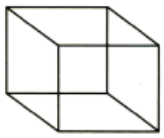
When we rotate rectangle about its axis, a cylinder is formed.

(Q) When a semicircle is rotated about its diameter, a sphere is formed.

(R) When a right angled triangle is rotated about its base, a cone is formed.



(S) Six square sheets are joined to form a cube



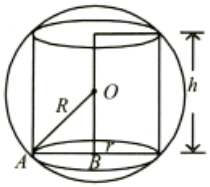
2. Match the lists correctly.

Column-I		Column-II	
(P)	A cylinder of radius 3 cm is inscribed in a sphere of radius 5 cm, then volume of cylinder is	(1)	38.5 m^3
(Q)	A conical pit of top diameter 3.5 m is 12 m deep, the capacity of pit is	(2)	512 cm^3
(R)	The length of a diagonal of a cube is $8\sqrt{3}$ cm, then volume of cube is	(3)	72 πcm^3
(S)	The capacity of a conical vessel with height 12 cm and slant height 13 cm is	(4)	100 πcm^3

	P	Q	R	S
(a)	4	3	2	1
(b)	1	3	4	2
(c)	3	1	2	4
(d)	3	2	4	1

Ans : (c) P − 3, Q − 1, R − 2, S − 4

(P)



Let height and radius of the cylinder be h and r respectively.
Let R be the radius of sphere.
In $\triangle OAB$

$$(OA)^2 = (OB)^2 + (AB)^2$$
$$R^2 = \left(\frac{h}{2}\right)^2 + r^2$$
$$R^2 - r^2 = \frac{h^2}{4}$$
$$h = 2\sqrt{R^2 - r^2}$$
$$= 2\sqrt{(5)^2 - (3)^2}$$
$$= 2\sqrt{25 - 9} = 2\sqrt{16}$$
$$= 2 \times 4 = 8\,\text{cm}$$
$$\text{Volume of cylinder} = \pi r^2 h$$
$$= \pi \times 3 \times 3 \times 8 = 72\pi\,\text{cm}^3$$

(Q) Let r, h be the radius and height of conical pit respectively.
Volume of conical pit

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times \frac{3.5}{2} \times \frac{3.5}{2} \times 12 = 38.5\,\text{m}^2$$

(R) Length of diagonal of cube of side

$$'a' = \sqrt{3}\,a$$
$$\sqrt{3}\,a = 8\sqrt{3}$$
$$a = 8\,\text{cm}$$
$$\text{Volume of cube} = (\text{side})^3 = 512\,\text{cm}^3$$

(S) Let V be the volume of conical vessel and h, r be the height & radius of vessel respectively.

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \pi \times \left(\sqrt{l^2 - h^2}\right)^2 \times 12$$
$$= \frac{1}{3} \times \pi \times \left[(13)^2 - (12)^2\right] \times 12$$
$$= \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi\,\text{cm}^3$$

3. Match the lists correctly :

Column-I		Column-II	
(P)	A cone and a hemisphere have equal base radius and equal volumes, then the ratio of height to radius is	(1)	1 : 9
(Q)	A cone and a cylinder are of the same height. Their radii of the bases are in ratio 2 : 1, then the ratio of their volumes is	(2)	1 : 1
(R)	A right circular cylinder just encloses a sphere of radius r , then the ratio of their curved surface areas is	(3)	4 : 3
(S)	Twenty seven solid iron spheres each of radius r and surface area S are melted to form a sphere with surface area S', then ratio of S and S' is	(4)	2 : 1

	P	Q	R	S
(a)	4	3	2	1
(b)	1	2	3	4
(c)	2	3	4	1
(d)	3	2	1	4

Ans : (a) P − 4, Q − 3, R − 2, S − 1

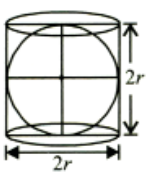
(P) $V_{\text{cone}} = V_{\text{hemisphere}}$ (r, h are the radius & height of cone & r be the radius of hemisphere)

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$
$$\Rightarrow \quad \frac{h}{r} = \frac{2}{1}$$
$$\Rightarrow \quad h : r = 2 : 1$$

(Q) Let h, r_1 and r_2 be the height, radius of cone and cylinder respectively.

$$\frac{r_1}{r_2} = \frac{2}{1}$$
$$V_{\text{cone}} = \frac{1}{3}\pi {r_1}^2 h;$$
$$V_{\text{cylinder}} = \pi r_2^2 h$$
$$\frac{V_{\text{cone}}}{V_{\text{cylinder}}} = \frac{\frac{1}{3}\pi {r_1}^2 h}{\pi r_2^2 h} = \frac{1}{3}\left(\frac{r_1}{r_2}\right)^2 = \frac{4}{3}$$
$$V_{\text{cone}} : V_{\text{cylinder}} = 4 : 3$$

(R)



$$\text{Curved surface area of sphere} = 4\pi r^2$$
$$\text{Curved surface area of cylinder} = 2\pi r(2r)$$
$$= 4\pi r^2$$

Ratio of curved surface area of sphere to that of

cylinder $= 4\pi r^2 : 4\pi r^2 = 1:1$

(S) Volume of 27 solid spheres

$=$Volume of a solid sphere

$$27 \times \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi r^3 \quad ...(1)$$

Now, $S = 4\pi r^2$ and $S' = 4\pi r'^2$

$$\frac{S}{S'} = \left(\frac{r}{r'}\right)^2$$

From (1)

$$27 = \frac{r'^3}{r^3}$$

$$\frac{r'}{r} = \frac{3}{1}$$

$$\frac{r}{r'} = \frac{1}{3}$$

$$\frac{S}{S'} = \frac{1}{9}$$

4. Match the lists correctly.

	Column-I		**Column-II**
(P)	Total surface area of right circular cylinder is	(1)	$\frac{2}{3}\pi r^3$
(Q)	Total surface area of right circular cone is	(2)	$\frac{4}{3}\pi r^3$
(R)	Total surface area of sphere is	(3)	$3 \times b \times h$
(S)	Total surface area of hemisphere is	(4)	$\frac{1}{3}\pi r^2 h$
(T)	Volume of cuboid is	(5)	$\sqrt{l^2+b^2+h^2}$
(U)	Volume of cylinder is	(6)	$\pi r^2 h$
(V)	Diagonal of cuboid is	(7)	$3\pi r^2$
(W)	Volume of sphere is	(8)	$4\pi r^2$
(X)	Volume of cone is	(9)	$\pi r(l+r)$
(Y)	Volume of hemisphere is	(10)	$2\pi r(r+h)$

Ans : P – 10, Q – 9, R – 8, S – 7, T – 3, U – 6, V – 5, W – 2, X – 4, Y – 1

5. Match the lists correctly.

	Column-I		**Column-II**
(P)	Volume of a cuboid is	(1)	perimeter of the base of the cylinder $\times h$
(Q)	Volume of a cube is	(2)	Area of circular base$\times$ height
(R)	Volume of a cylinder is	(3)	$(\text{edge})^3$
(S)	Curved surface area of the cylinder is	(4)	base $\times$ area $\times$ height
		(5)	$2\pi rh$
		(6)	length $\times$ breadth $\times$ height
		(7)	edge $\times$ edge $\times$ edge
		(8)	$\pi r^2 h$

Ans : P – (4, 6), Q – (3, 7) R – (2, 8), S – (1, 5)

5. ASSERTION AND REASON

DIRECTION : In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

1. **Assertion :** The external dimensions of a wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of the wood is 15 mm, then the internal volume is $765\,\text{cm}^3$.

Reason : If external dimensions of a rectangular box be l, b and h and the thickness of its sides be x, then its internal volume is $(l-2x)(b-2x)(h-2x)$.

Ans : (a) Both assertion and reason are true and reason is the correct explanation of assertion.

$$\text{Length of box} = 18\,\text{cm}$$

$$\text{Width of box} = 10\,\text{cm}$$

$$\text{Height of box} = 6\,\text{cm}$$

$$\text{thickness of box} = 5\,\text{mm} = \frac{1}{2}\,\text{cm}$$

Internal length, width, height of the box is

$$\left(18 - \frac{2\times 1}{2}\right), \left(10 - \frac{2\times 1}{2}\right), \left(6 - \frac{2\times 1}{2}\right)$$

$$\text{Internal volume of box} = 17 \times 9 \times 5 = 765\,\text{cm}^3$$

2. **Assertion :** A cone is a solid figure.

Reason : A cone is generated when rectangular sheet is rotated about its axis.

Ans : (c) Assertion is true but reason is false.

3. **Assertion :** If the inner dimensions of a cuboidal box are $50\,\text{cm} \times 40\,\text{cm} \times 30\,\text{cm}$, then the length of the longest rod that can be placed in the box is $50\sqrt{2}$ cm.

Reason : The line joining opposite corners of a cuboid is called its diagonal.

Also, length of longest rod = length of diagonal.

$= \sqrt{l^2+b^2+h^2}$

Ans : (a) Both Assertion and Reason are correct and

Reason is the correct explanation of Assertion.

Length of the diagonal is the length of the longest rod that can be placed in the cuboid.

4. **Assertion :** Into a circular drum of radius 4.2 m and height 3.5 m, the number of full bags of wheat can be emptied, if the space required for wheat in each bag is 2.1 cu. m, is 92 bags nearly.

Reason : Volume of circular drum is $\frac{1}{3}\pi r^2 h$ where r is radius and h is height of drum.

Ans : (c) If assertion is true but reason is false.

Radius of circular drum, $r = 4.2\,\text{m}$

Height of circular drum, $h = 3.5\,\text{m}$

Volume of circular drum

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 4.2 \times 4.2 \times 3.5\,\text{m}^3$$

Space occupied by 1 wheat bag$= 2.1\,\text{m}^3$

Number of wheat bags

$$= \frac{\text{Volume of drum}}{\text{Space occupied by 1 bag}}$$

$$= \frac{\frac{22}{7} \times 4.2 \times 4.2 \times 3.5}{2.1}$$

$$= 92.4 = 92 \text{ nearly}$$

5. **Assertion :** An edge of a cube measures $r\,\text{cm}$. If the largest possible right circular cone is cut out of this cube, then the volume of the cone is $\frac{1}{6}\pi r^3$.

Reason : Volume of the cone is given by $\frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height of the cone.

Ans : (d) Assertion is incorrect but Reason is correct.

6. **Assertion :** A shot put is a metallic sphere of radius 4 cm. If the density of the metal is 10 g per cm^3. then the mass of the shot put is 2 kg.

Reason : Volume of sphere of radius r is $\frac{4}{3}\pi r^3$.

Ans : (d) Assertion is false but reason is true.

$$\text{Volume of metallic sphere} = \frac{4}{3}\pi(4)^3\,\text{cm}^3$$

$$\text{Density of metal} = 10\text{g/cm}^3$$

$$\text{Mass of shot put} = \text{Density} \times \text{Volume}$$

$$= 10 \times \frac{4}{3}\pi(4)^3$$

$$= 2681.90\,\text{g}$$

$$= 2.68\,\text{kg} \neq 2\,\text{kg}.$$

7. **Assertion :** In a cylinder, if radius is halved and height is doubled, the volume will be halved.

Reason : In a cylinder, radius is doubled and height is halved, curved surface area will be same.

Ans : (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

8. **Assertion :** The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is $(\pi)\,r\left(l+\frac{r}{4}\right)$.

Reason : Total surface area of cone is $\pi r(l+r)$ where r is radius and l is the slant height of the cone.

Ans : (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

9. **Assertion :** If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Reason : Curved surface area is increased when diameter decreases.

Ans : (c) Assertion is true but reason is false.

Let the diameter of sphere be $2r$.

$$\text{Decreased diameter} = \left(2r - \frac{25}{100} \times 2r\right)$$

$$= 2r - \frac{r}{2} = \frac{3r}{2}$$

$$\text{New diameter} = \frac{3r}{2}$$

$$\text{and Radius, } r = \frac{3r}{4}$$

Curved surface area of sphere

$$= 4\pi r^2$$

$$\text{New curved surface area} = 4\pi\left(\frac{3r}{4}\right)^2 = 4\pi\left(\frac{9r^2}{16}\right)$$

$$\text{Difference} = 4\pi r^2\left(\frac{9}{16} - 1\right)$$

$$= -4\pi r^2\left(\frac{7}{16}\right)$$

$$= -4\pi r^2\left(\frac{7}{16} \times 100\right)\frac{1}{100}$$

$$= 4\pi r^2(-43.75)\%$$

Curved surface area is decreased by 43.75%.

As if diameter is decreased, curved surface area is decreased.

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